For FSU, Jan. 2018, D. Huse, Lecture 2.

Many-body localization (MBL).

basic review:
Nandkishore + H.
Ann. Rev. C. M. Phys. (2015)

is Anderson localization with interactions in highly-excited states.

Systems in MBL phase do not thermalize.

A new type of quantum phase transition: MBL-to-thermal.

The MBL phase is a type of "quantum memory": (some) local memory of local initial state to long time limit.

"Localization-protected order": MBL can dynamically stabilize long-range and/or topological order in the initial state that would be absent at thermal equilibrium.

One example: "Discrete time crystals".

Simplest example, spin-1/2 chain:

$$H = \sum_n h_n Z_n$$ (we will add hoppings + interactions) soon

(could also do Floquet MBL)

this also a particle model: $\downarrow$ = no particle
$\uparrow$ = fermion or hard-core boson.

e.g. cold atoms in an irregular lattice.

$[H, Z_n] = 0$: all Z_n are localized conserved operators.

Eigenstates are any pattern of $\uparrow$'s and $\downarrow$'s.

For stability of this trivial MBL to hoppings + interactions, want $h_n \neq h_m$:
few resonances: examples: "detuning"

- h_n random from continuous $P(h)$
- $h_n = h\cos(qn + \phi_0)$ <u>nonrandom</u> quasiperiodic, $\frac{q}{2\pi}$ irrational

$$H = \sum_n h_n Z_n + \underline{\underline{\text{weak}}} \text{ short-range hoppings + interactions in } \underline{\underline{1D}}$$

For random cases: (BAA)

perturbative stability of MBL: Basko, Aleiner, Altshuler 2006

nonperturbative stability of MBL $\underline{\underline{\text{in 1D}}}$: Imbrie '14

Quasiperiodic: BAA argument works, numerics (Khemani, Sheng, H. '17)

show MBL more stable for quasiperiodic than for random.

Interactions require localized conserved operators to be "dressed"

"l-bits":

$$\tau_n = Z_n + \text{"dressing"}$$

contains multisite terms that have typical magnitude + probability of being large that fall exponentially with distance.

$$[\tau_m, \tau_n] = [H, \tau_n] = 0$$

Pauli algebra is preserved.

MBL is a type of integrability (but $\underline{\underline{\text{not}}}$ fine-tuned).

MBL phase survives weak interactions, but may thermalize for strong interactions (or too long range)

H in terms of the l-bit operators: is an Ising model: $\tau_n = \pm 1$

$$H = \sum_n \tilde{h}_n \tau_n + \sum_{m,n} J_{m,n} \tau_m \tau_n + \sum_{l,m,n} K_{lmn} \tau_l \tau_m \tau_n + \cdots$$

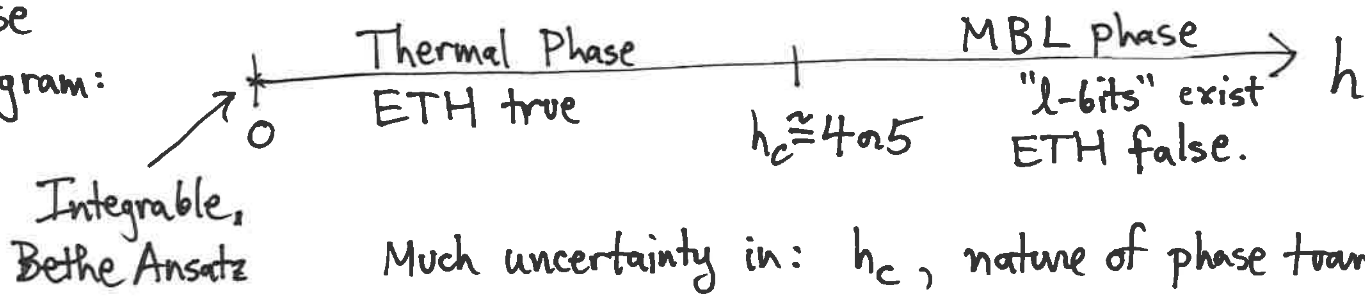
interactions: fall off exponentially with distance
do not contain any l-bit flip operators.

Phase transition to thermal phase: l-bits delocalize.

"Standard model" of MBL is a spin-1/2 chain:

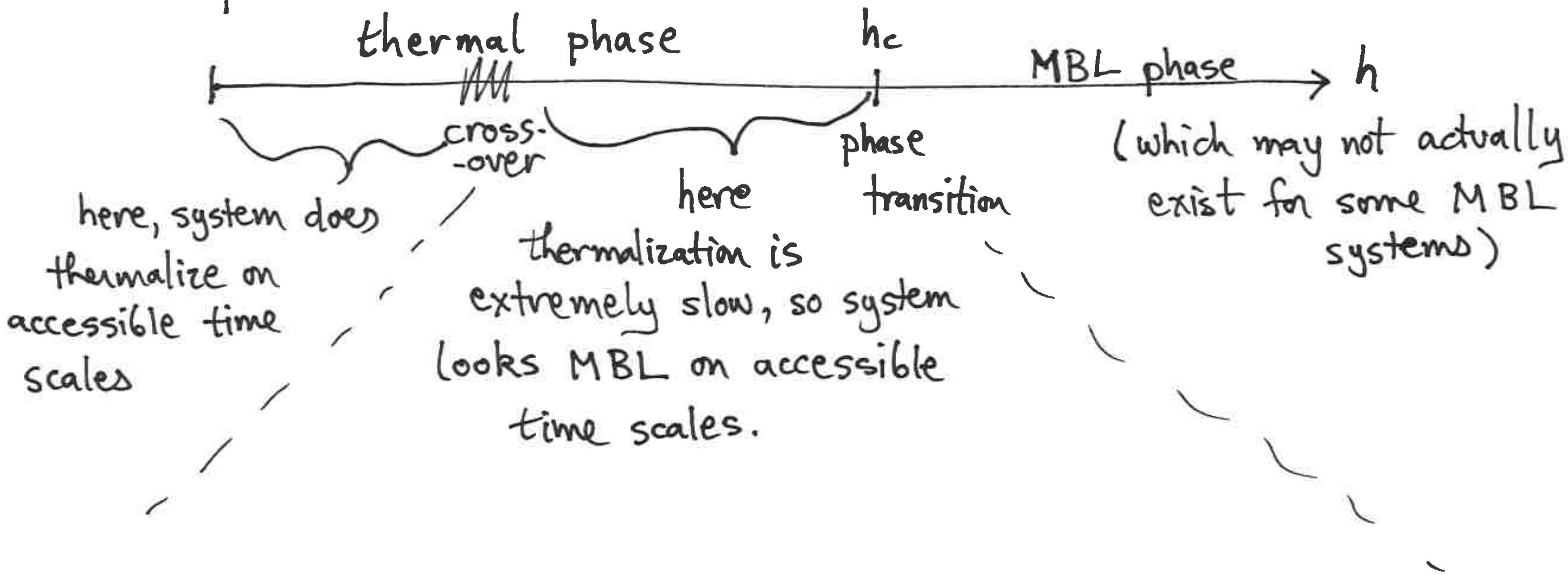
$$H = \sum_n \left[h_n S_n^z + \vec{S}_n \cdot \vec{S}_{n+1} \right] \qquad h_n \text{ random, uniform in } [-h, h]$$

Phase Diagram:



Much uncertainty in: h_c, nature of phase transition, & could there be an intermediate phase?

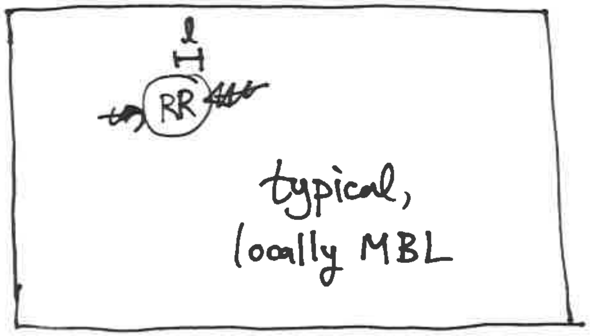
In practice:



Open question: Can there be an actual intermediate phase (not ETH, no l-bits)?
Or is it "just" a very slow part of the thermal phase?

Nonperturbative rare region (Griffiths) effects. Gopalakishnan, et al. 1511.06389, 1502.07712

In the MBL phase: Random case in d dimensions.



RR: rare region of radius l where, by chance, randomness is weak. Such regions do occur for any finite l in an infinite system. their density $\sim \exp(-cl^d)$

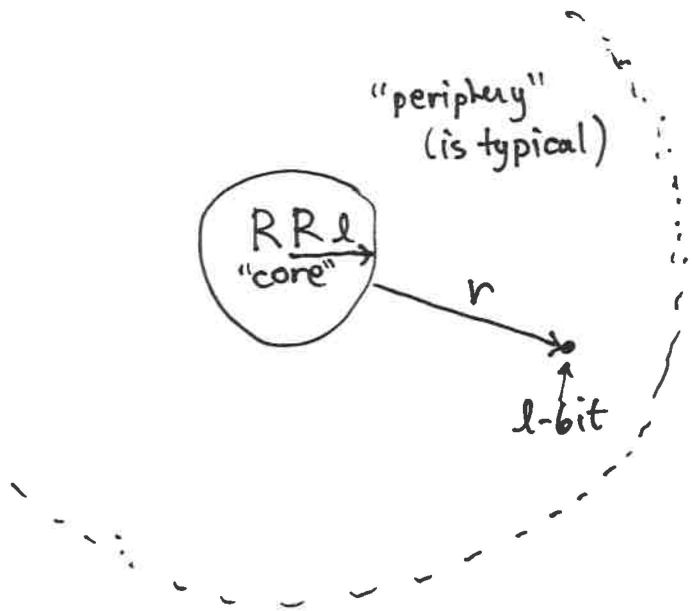
If we ignore couplings to RR, can make l-bits in all "typical" regions.

RR contains $\sim l^d$ degrees of freedom,

has many-body level spacing $\delta \sim \exp(-l^d)$ (with some length unit)

for large l, acts as a local thermal bath.

Consider an l-bit at distance r from the RR:



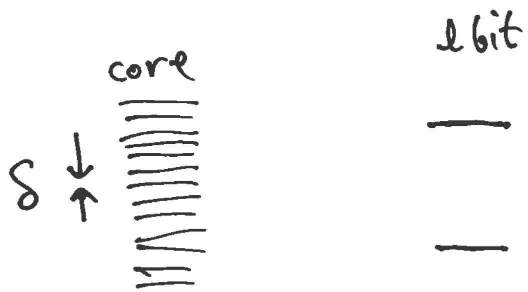
Only "core" has weak randomness.

Coupling RR $\longleftrightarrow$ ℓ-bit has matrix elements

$$\sim \exp(-r/(2\xi)) \text{ for } \ell\text{-bit flips.}$$

decay length of ℓ-bit "dressing"

Fermi Golden Rule estimate of ℓ-bit lifetime:



$\sim \exp(-r/\xi)$ is valid when $\gg \delta \sim \exp(-\ell^d)$

then ℓ-bit gets entangled with RR, so effective bath has radius $(\ell + r)$

Size of periphery is set by:

$$\exp(-r/\xi) \sim \exp(-(r+\ell)^d)$$

$$r \sim \xi (r+\ell)^d$$

1D, for $\xi < 1$:

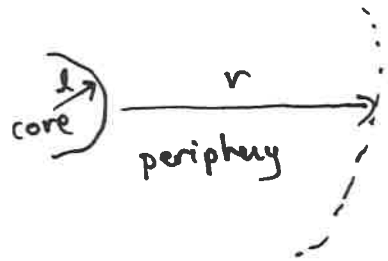
$$r = \frac{\xi}{1-\xi}\,\ell \text{ is finite}$$

1D MBL phase can remain stable, when $\xi < 1$. Imbrie '14

1711.09880

Is destabilized (phase transition), when $\xi \to 1$. Thiery, Müller, de Roeck

RR core of radius ℓ $\Rightarrow$ periphery set by $\quad r \sim \zeta (r+\ell)^d$



For $d>1$ and large finite ℓ: no solution:

RR thermalizes (slowly) the whole system:

No true MBL phase with randomness in $d>1$.

De Roeck + Huveneers 1608.01815

But, for nonrandom (e.g. quasiperiodic) "detunings" the MBL phase may be able to survive in $d>1$ (no rare regions).

In 1D, numerics shows MBL phase is more stable and less entangled for quasiperiodic as compared to random.

Khemani, Sheng, H, PRL '17

Difference is probably due to rare region effects.

MBL-to-thermal quantum phase transition.

<u>Not</u> a conventional ground-state quantum phase transition.

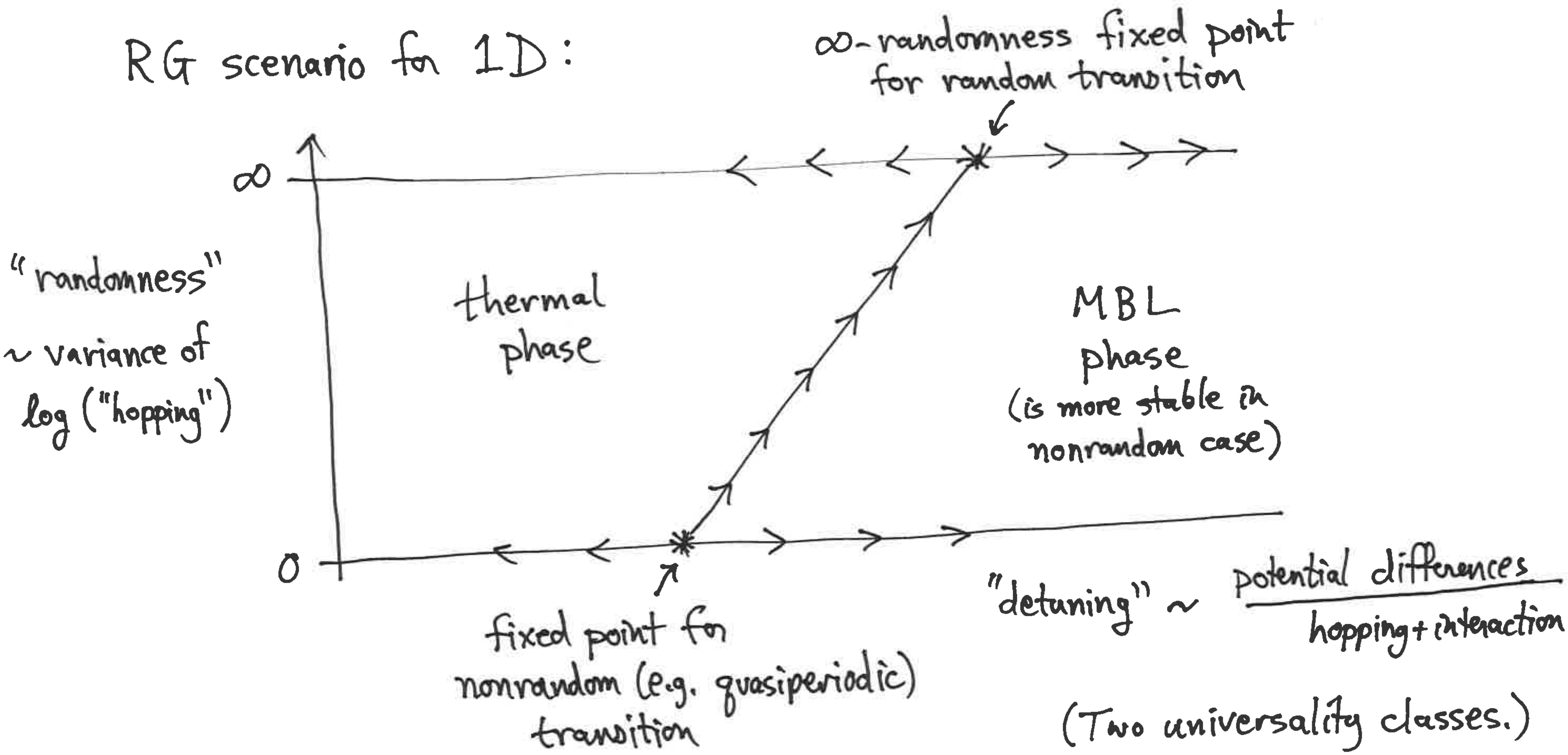
Incompletely understood.

Appears to be some "hybrid" of continuous and discontinuous.

Properties of this transition in 1D are studied by:

- numerics, mostly exact diagonalizations
- approximate strong-randomness renormalization group
- usually looking at <u>all</u> states, thus <u>infinite temperature.</u>

RG scenario for 1D:



added randomness is
RG-relevant here. Khemani, Sheng, H PRL '17.

Flow towards stronger randomness is clearly seen in
finite-sized data: growing sample-to-sample differences.

Khemani, et al. PRX '17.

Strong randomness RG approximations

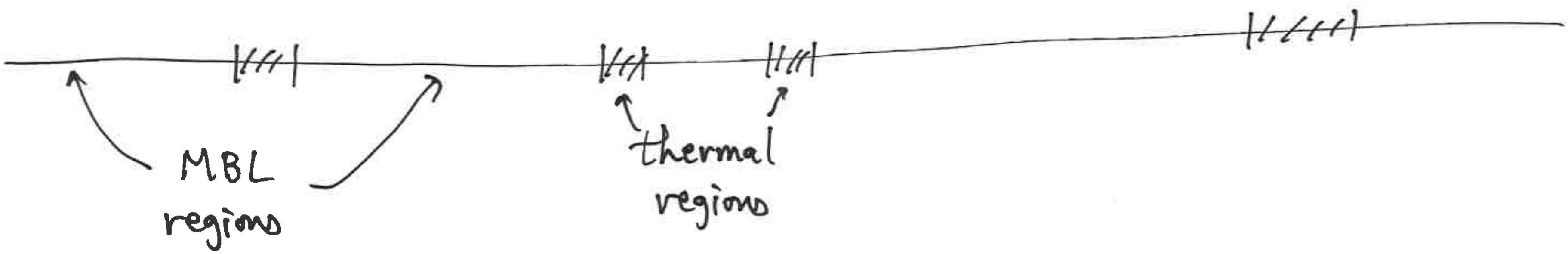
→ One dimension ←

Vosk, H, Altman 1412.3117
Dumitrescu, Vasseur, Potter 1701.04827
Thiery, Müller, De Roeck 1711.09880

Look at system's local dynamics on energy scale Γ_0 $(\hbar = 1)$:

MBL regions have "provisional" ℓ-bits with decay rate $< \Gamma_0$

Thermal regions do not: they have many-body level spacing $\delta < \Gamma_0$
so local spectrum looks continuous on this scale.



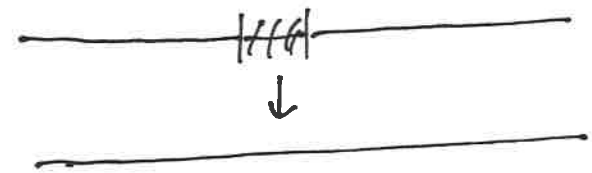
Near transition, system is mostly MBL.

Have distributions of lengths of thermal regions,
couplings between thermal regions and ℓ-bits.

Strong randomness

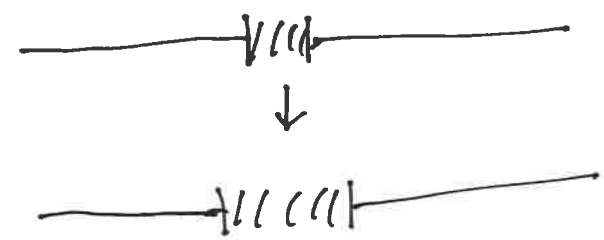
Now "flow" to lower energy scales $\Gamma \to \Gamma' < \Gamma_0$

What can happen?

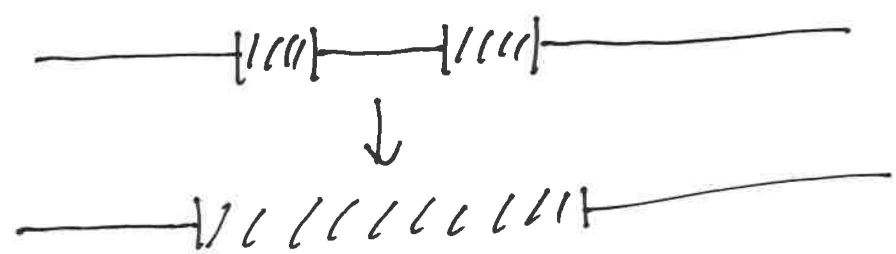


1) Thermal region has $\Gamma' < \delta < \Gamma_0$, is lost, becomes localized on scale Γ'.

2) Nearby l-bits "decay" by coupling to thermal region, which thus grows



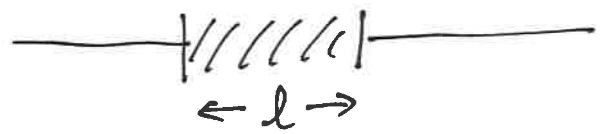
3) Two (or more) thermal regions connect and merge:



MBL phase: loss dominates.

Thermal phase: one thermal region grows to cover entire system.

(Note: it only takes one: the MBL phase is "fragile")

How long ℓ  does a thermal region grow under coarse-graining before it has $\delta > \Gamma$ and gets localized? This is the local entanglement length.

In MBL phase: $P(\ell) \sim \exp(-c\ell^{\beta})$ $\quad \beta < 1$

"stretched exponential"

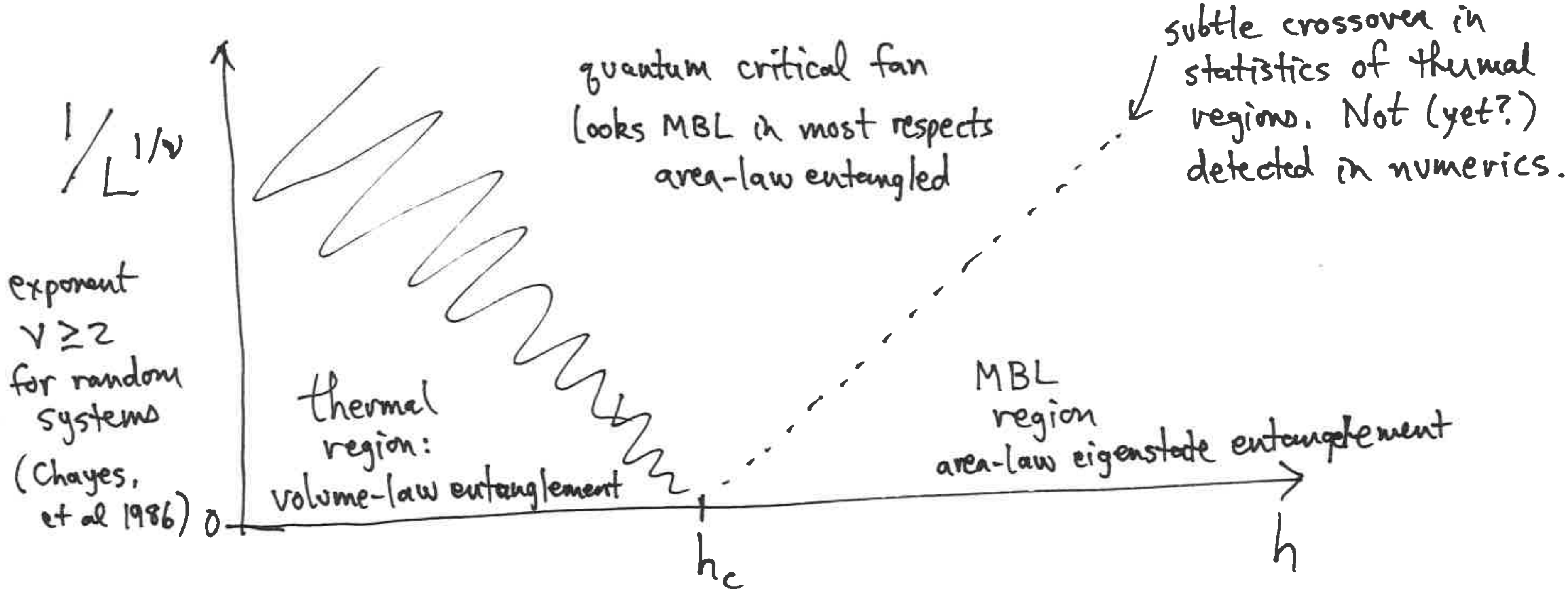
In critical regime at transition: power-law distribution of ℓ:

$$P(\ell) \sim \ell^{-\gamma} \qquad \gamma > 2$$

thermal regions occupy a small fraction of system, typical locations remain MBL, area-law entangled.

In thermal phase $P(\ell) \sim \delta_{\ell, L}$ a thermal region grows to cover the full system.

Finite-size properties: Systems of length L
(from RG and from numerics)



Almost all of phase transition happens on thermal side of transition, when samples get large enough to have a large thermal region that destabilizes MBL.

On approach from MBL phase at large L, transition is in many respects discontinuous (an instability to thermalization).

SUMMARY

Many-body localization

- A type of integrability: localized conserved operators.

  Many open questions remain.

- Nonperturbative rare region effects: may destabilize MBL phase for $d > 1$.

- MBL–thermal phase transition:

  Two universality classes in 1D.

  RG picture of transition in random systems.