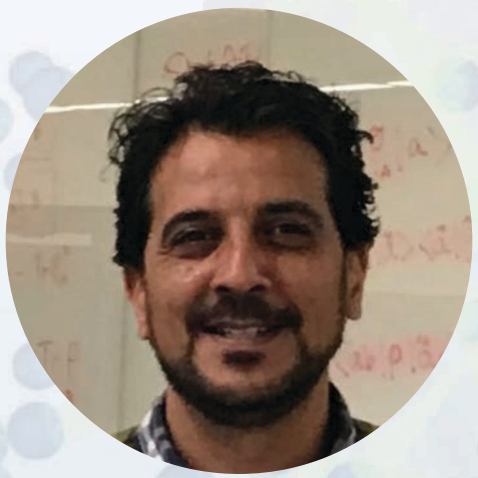
ENTANGLEMENT IN QUANTUM LIQUIDS & GASES

Understanding how quantum information is encoded in quantum matter

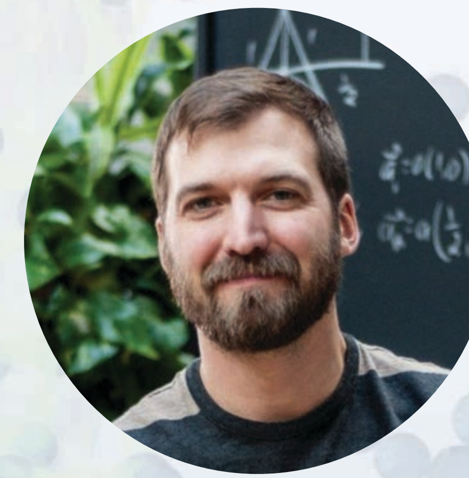


H. Barghathi
UVM

E. Casiano-Diaz
UVM

C. Herdman
UVM → IQC → Middlebury

P.-N. Roy
U Waterloo



R.G. Melko
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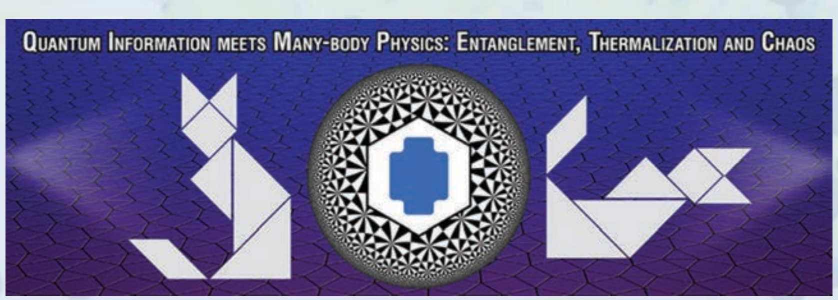

Adrian **Del Maestro**
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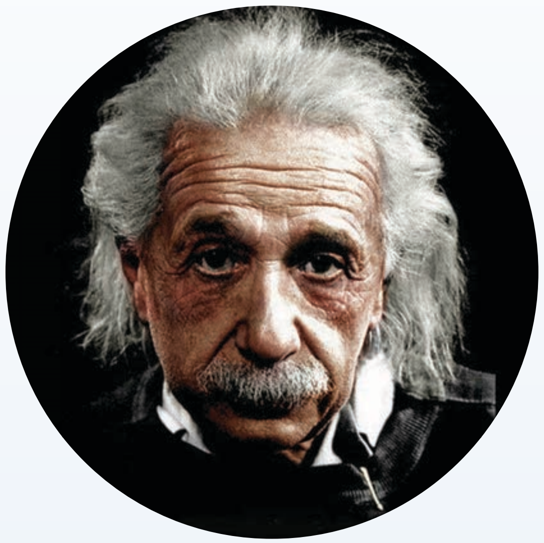
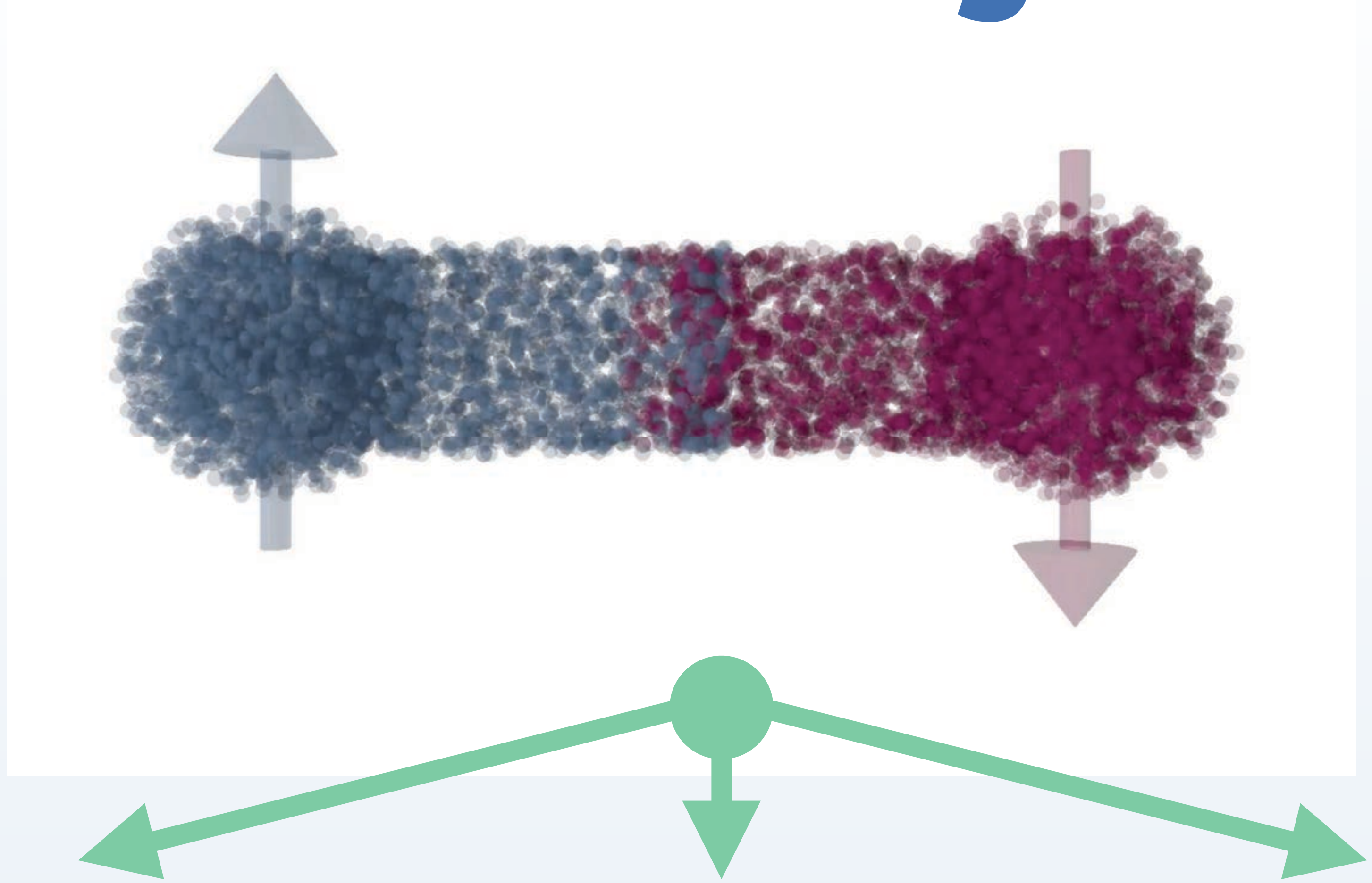
Quantum Entanglement

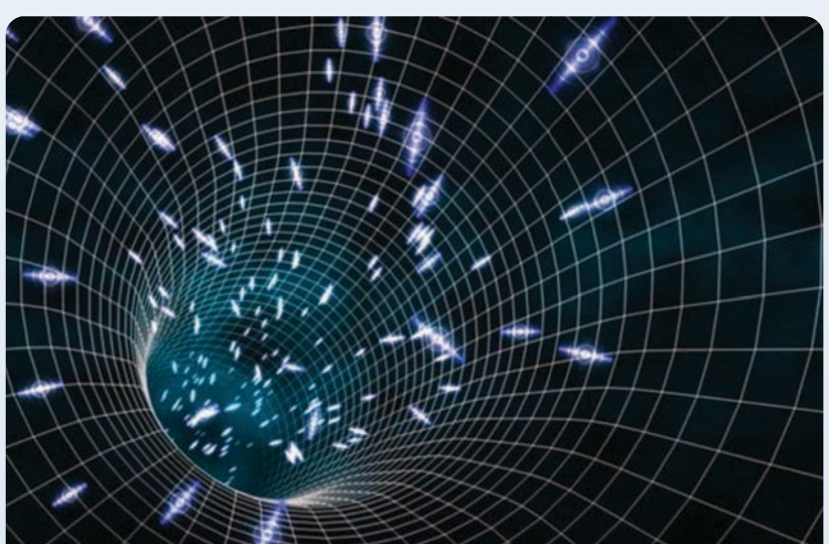


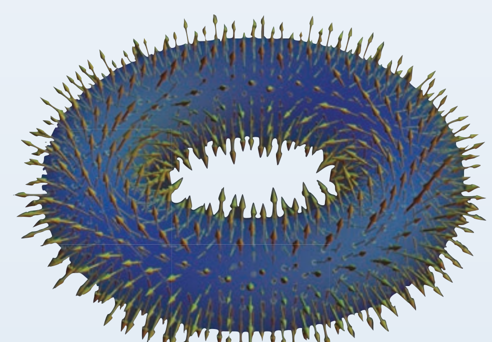


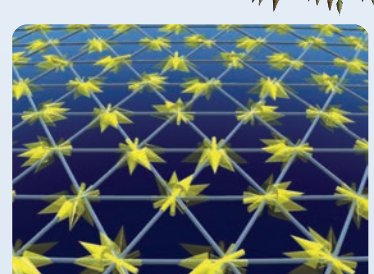
emergent
spacetime

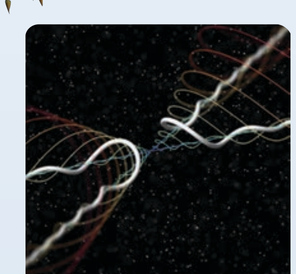
quantum
matter

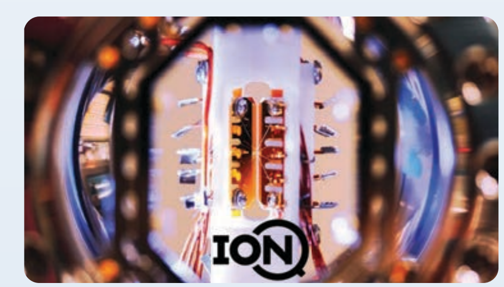
quantum
technologies

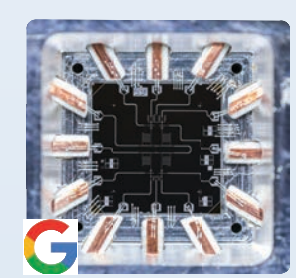


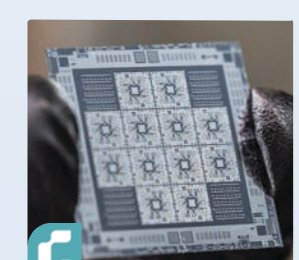










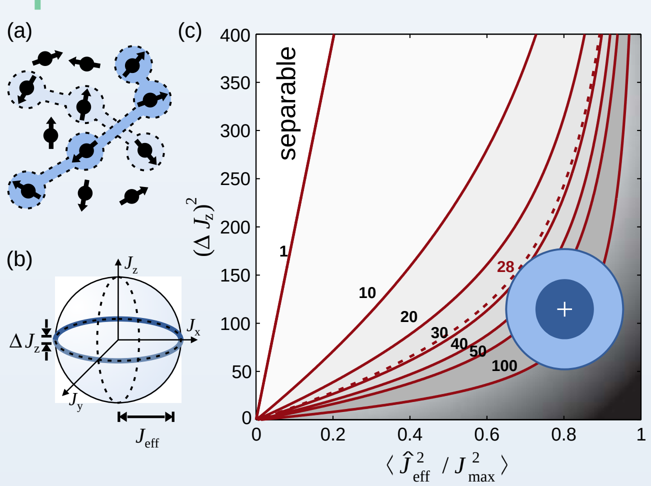


Quantum liquids and gases

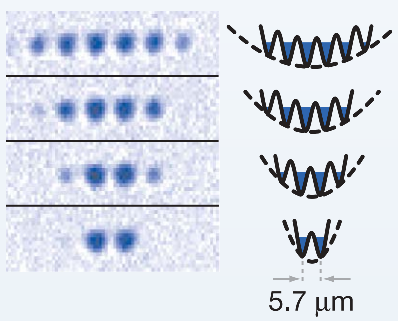
Theory: focused on systems with discrete Hilbert spaces with local degrees of freedom: qubits, insulating lattice models, …

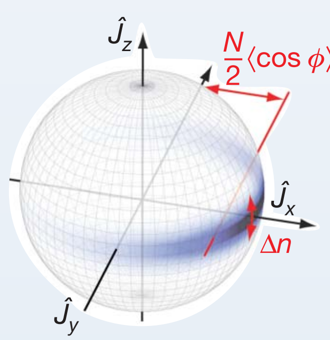
Experiments employ the quantum and itinerant positional states of ultra-cold atomic gasses and BECs

observation and manipulation of Dicke states



B. Lücke, *et.al.*, PRL 112, 155304 (2014)



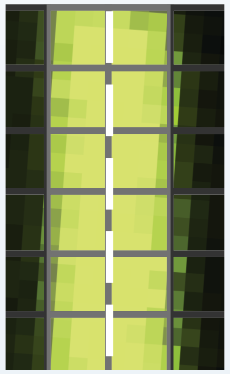


ultra high-precision quantum interferometry

.Estève, *et al.*,
Nature 455, 1216 (2008)

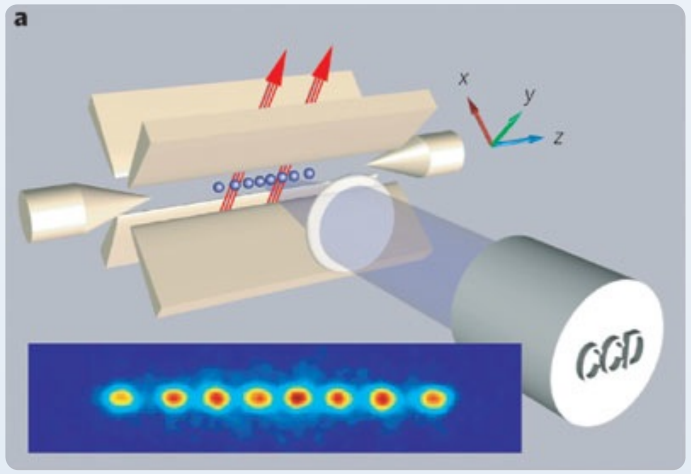
Rényi entropy in lattice gases

R. Islam, *et.al.*, Nature (2015)



boson sampling

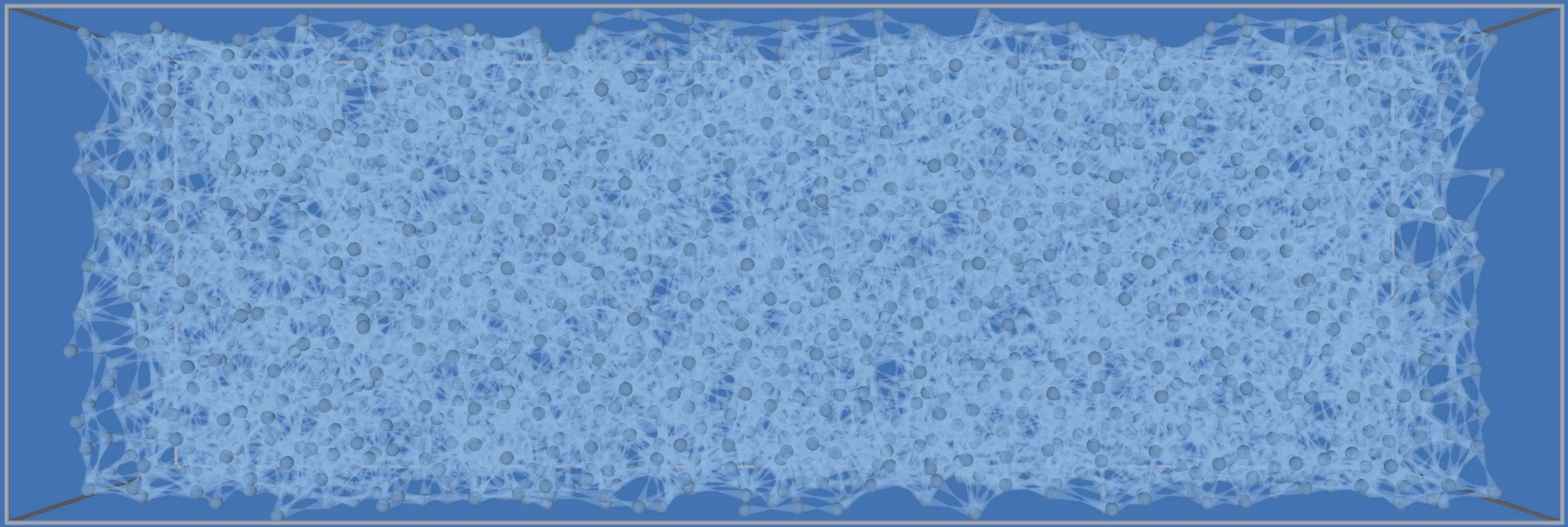
C. Shen, *et al.*, PRL 112, 050504 (2014)



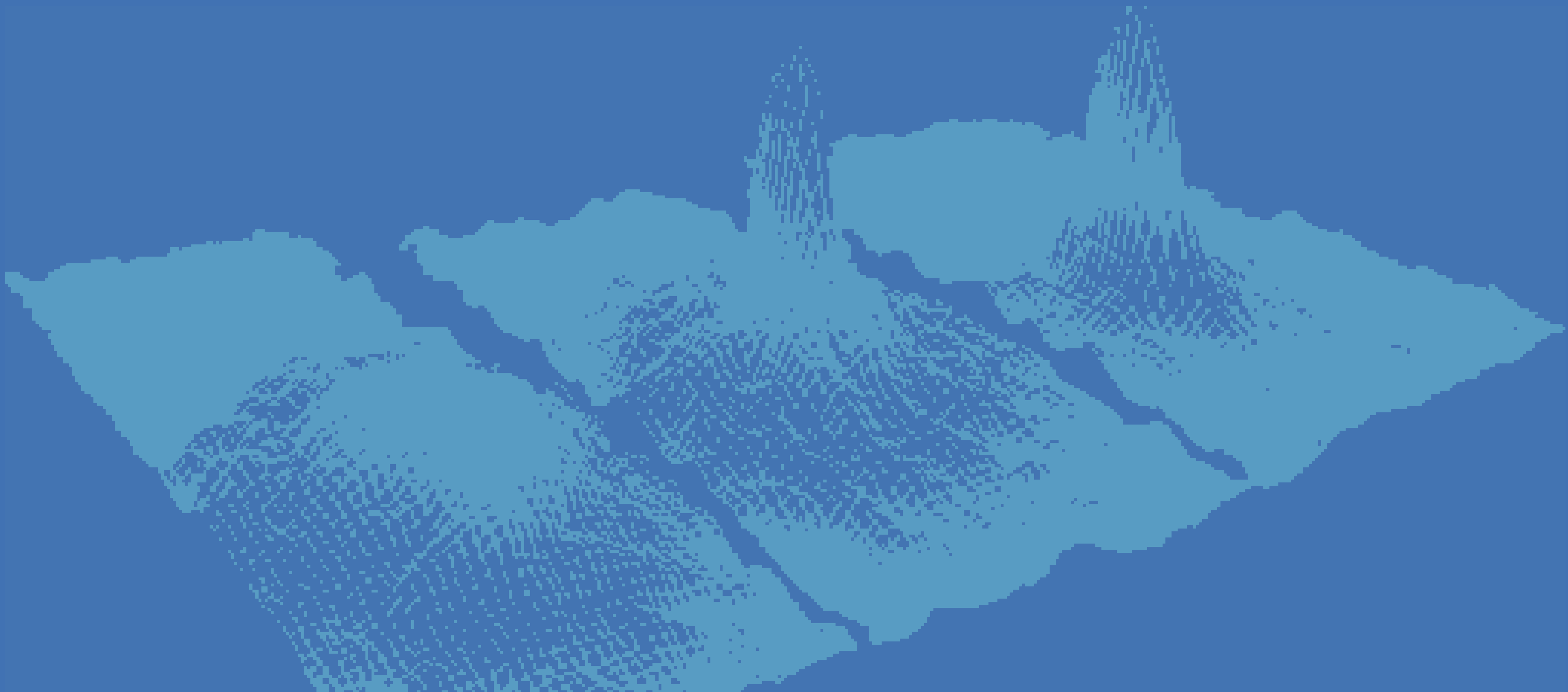
multiparticle entanglement of trapped ions

T. Monz, *et.al.*, PRL 102, 040501 (2009)

How does quantum *indistinguishability* affect entanglement?

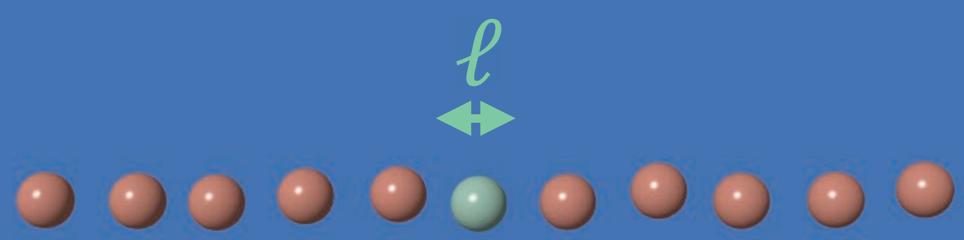


Can we use the *entanglement* in quantum fluids as a *resource* for information processing?

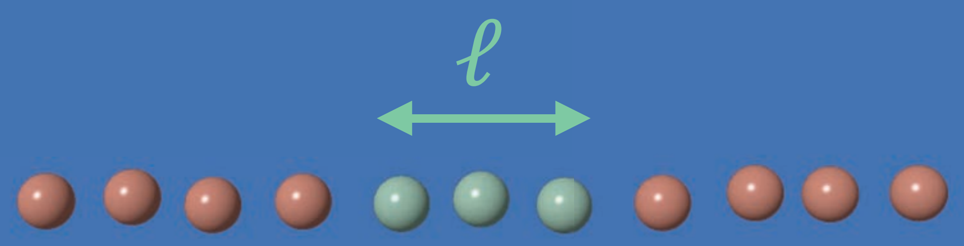


How does entanglement scale with the size of the subregion?

How does entanglement *scale* with the size of the subregion?



How does entanglement *scale* with the size of the subregion?



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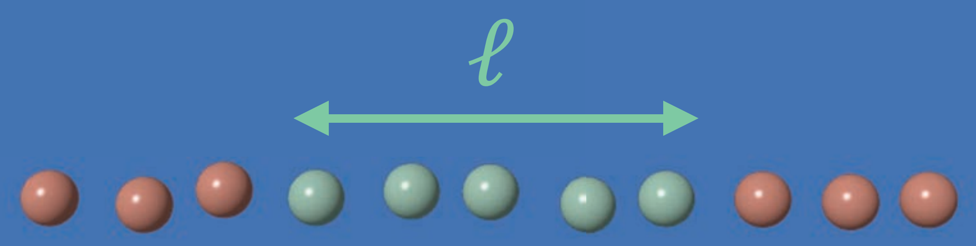

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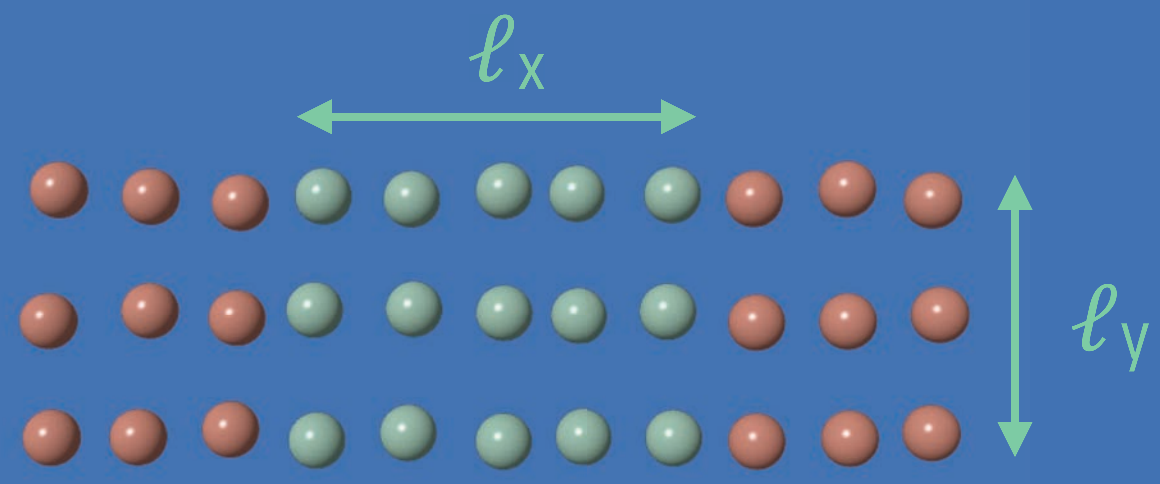

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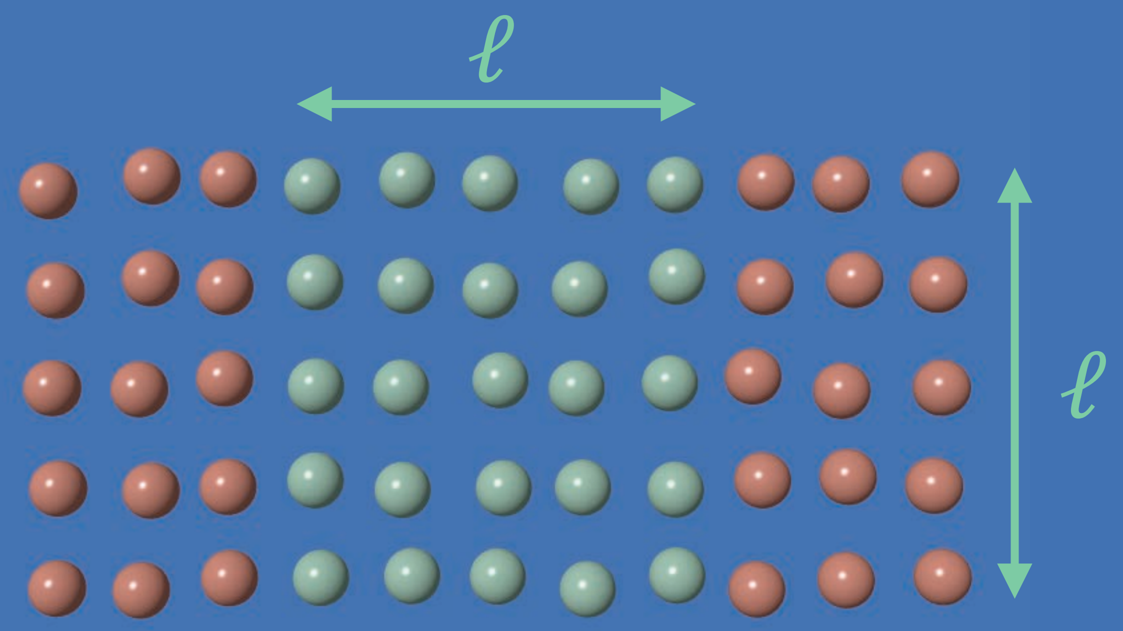

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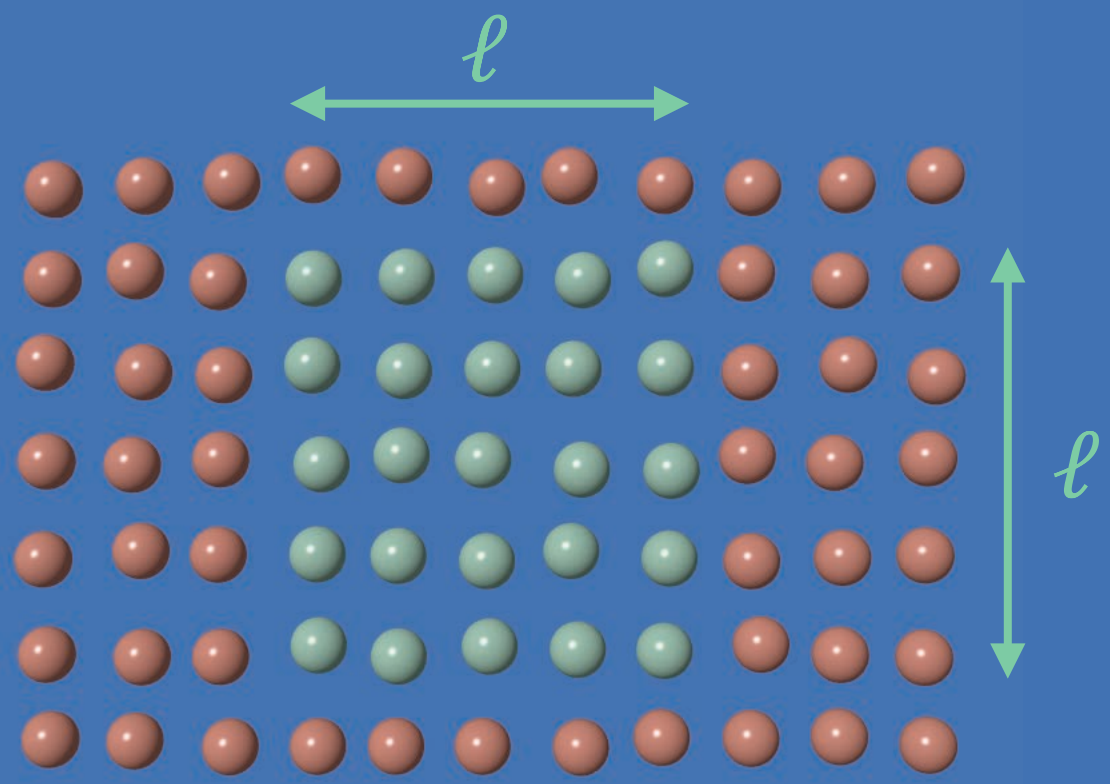

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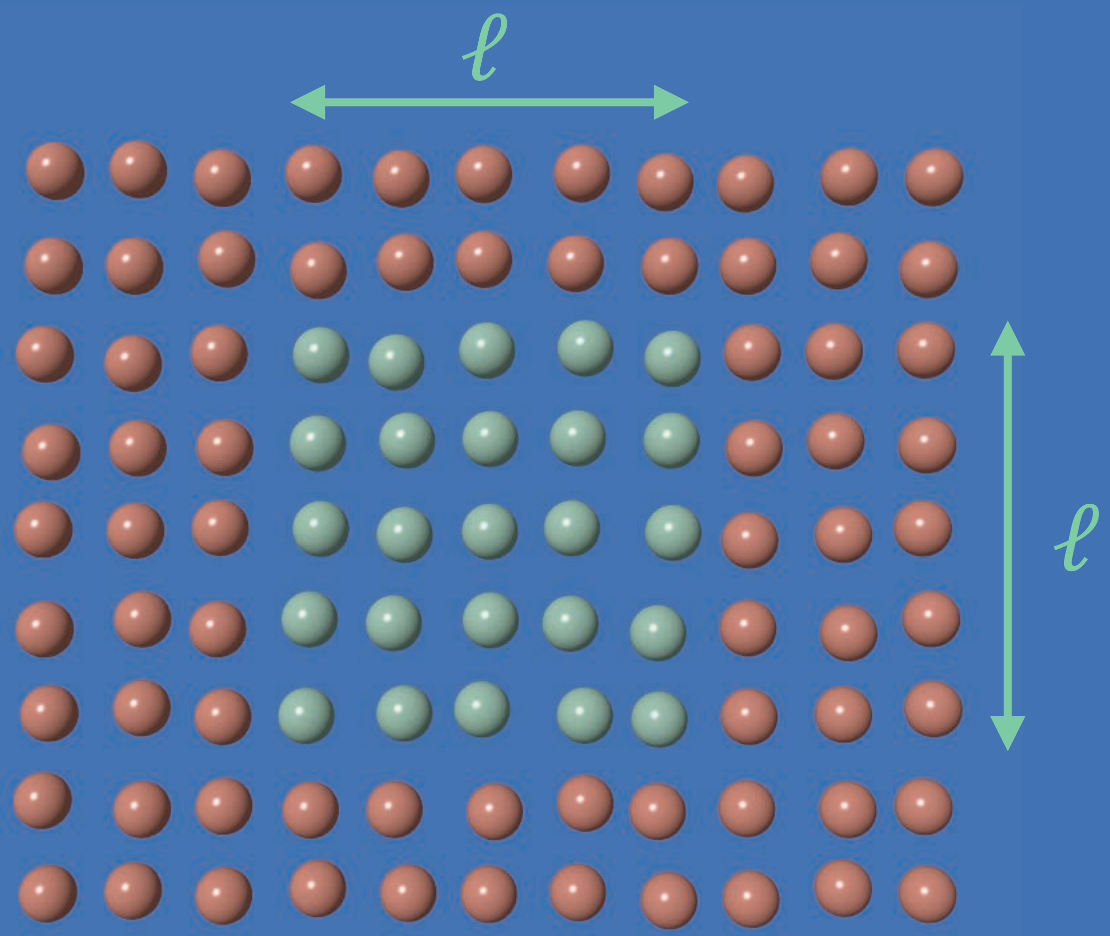
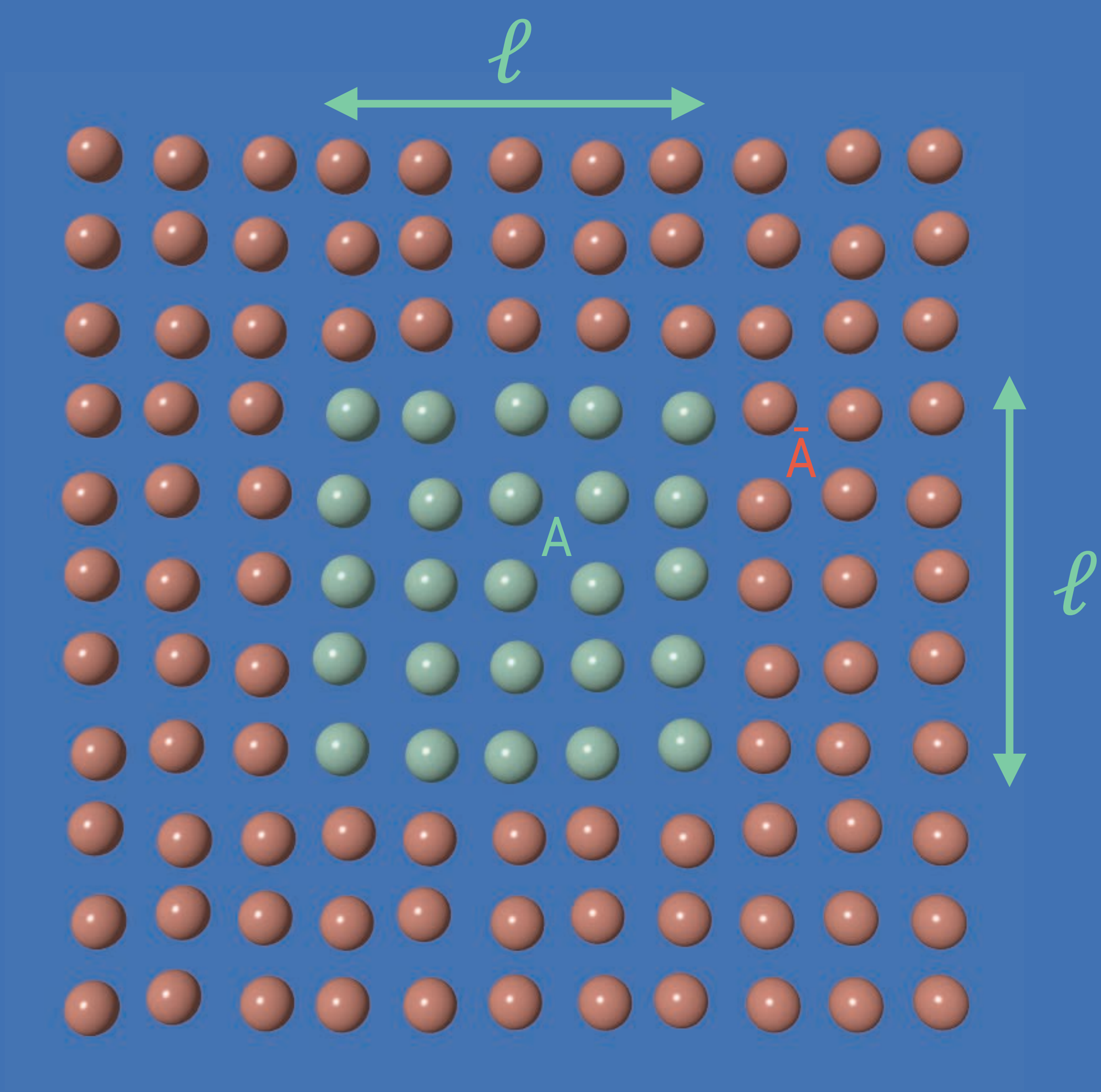


How does entanglement *scale* with the size of the subregion?

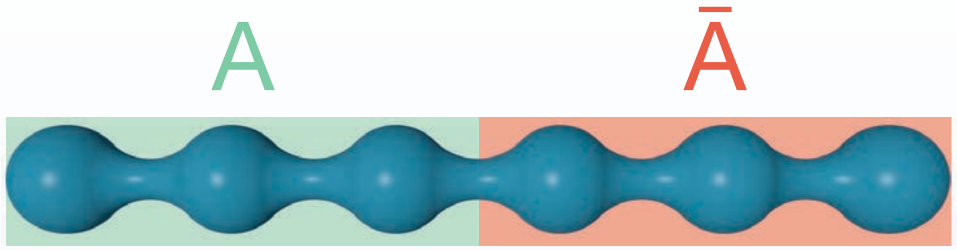


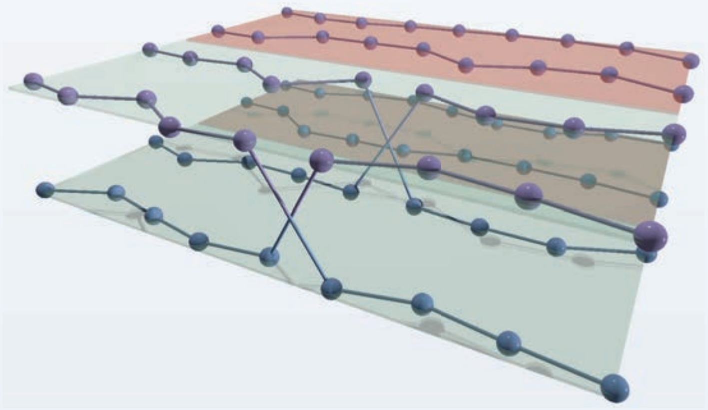
$$S(\ell) \sim \ell^{\lambda}$$

$$\lambda = ?$$

Entanglement and Entropy

quantifying uncertainty in many-body systems



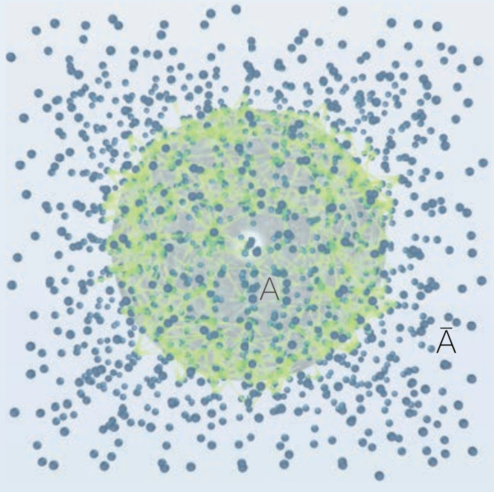


Measuring Entanglement

SWAP algorithm in experiment and quantum Monte Carlo

Results for Quantum Liquids & Gases

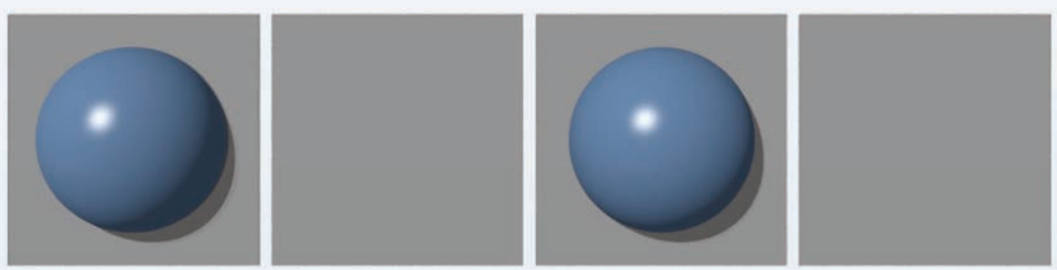
benchmarking, scaling and the area law



Toy Quantum Matter

bosons with hard-cores on a 1d lattice

L = 4 sites



N = 2 bosons

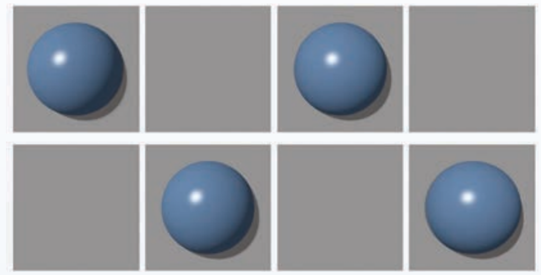
kinetic potential

$$H = -\sum_i \left(b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i \right) + V \sum_i n_i n_{i+1}$$

Investigate the quantum ground state for different interaction strengths V

What are the ground states?

$V \gg 1$



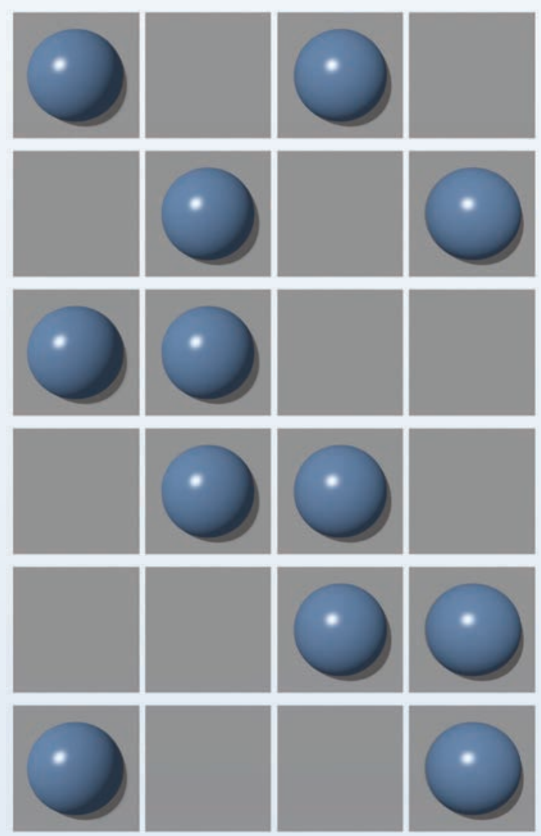
solid

$$T = -\sum_i \left(b_i^\dagger b_{i+1} + h.c.\right)$$

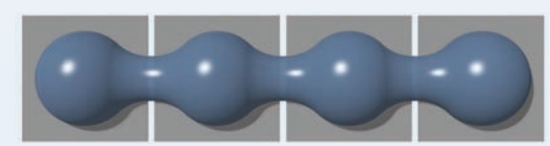
$$U = V \sum_i n_i n_{i+1}$$

$$|\Phi\rangle = \frac{1}{\sqrt{2}} \left(|1010\rangle + |0101\rangle\right)$$

$V \ll 1$



$\equiv$

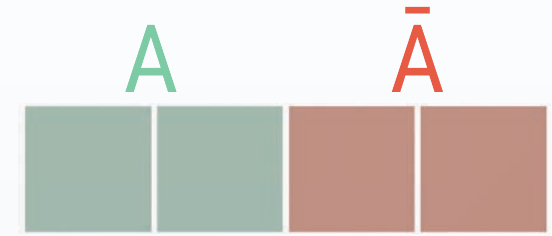


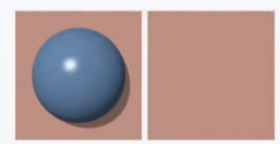
superfluid

$$|\Psi\rangle = \frac{1}{2} \left(|1010\rangle + |0101\rangle\right) + \frac{1}{2\sqrt{2}} \left(|1100\rangle + |0011\rangle + |1001\rangle + |0110\rangle\right)$$

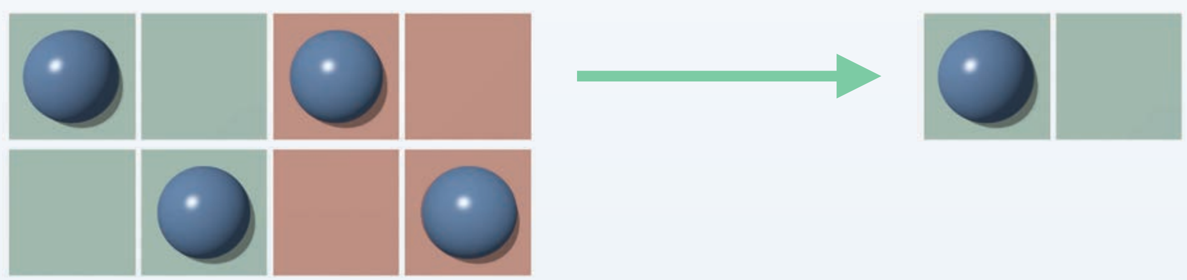
A Quantum Bipartition

Break up the system into two parts
and make a local measurement on $\bar{A}$



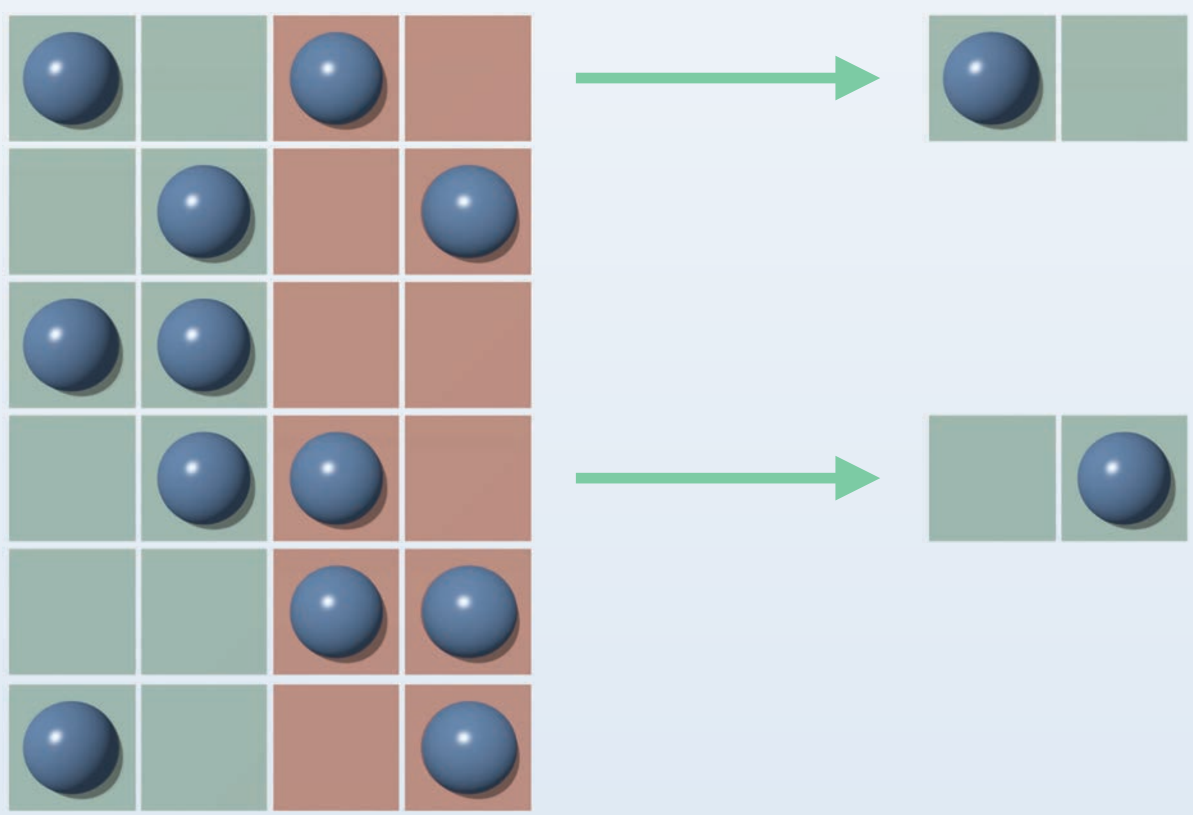
Suppose we find:  what do we know about A?

$V \gg 1$



no uncertainty = complete knowledge

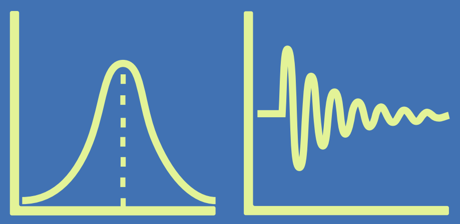
$V \ll 1$



uncertainty = incomplete knowledge

Entanglement

quantum information that is encoded non-locally in the joint state of a system

- Can it be quantified? 
- Can it be measured? 

Quantifying Entanglement with Entropy

Entropy: A measure of encoded information

Entanglement: Non-locally encoded quantum information

Entanglement Entropy: A measure of entanglement

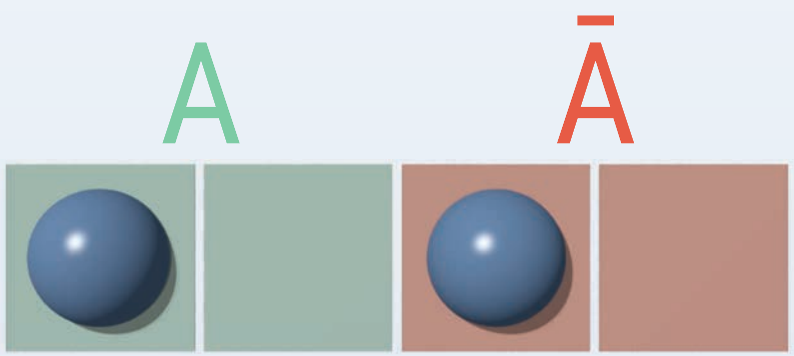
probabilities encoded in reduced density matrix

Shannon

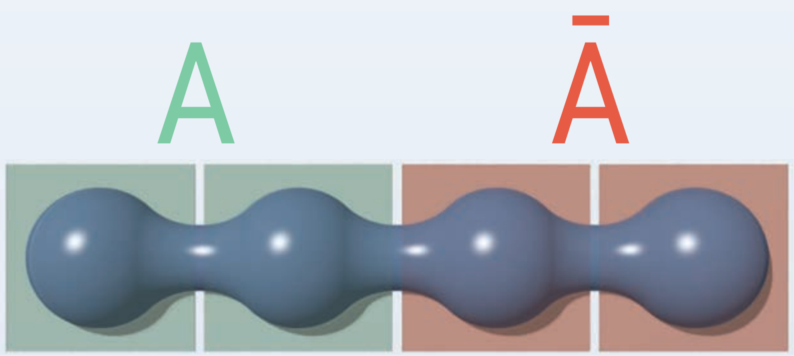
$$S = -\sum_a p_a \log p_a$$

$$S(A) = -Tr \rho_A \log \rho_A$$

von Neumann



S(A) = 0



S(A) > 0

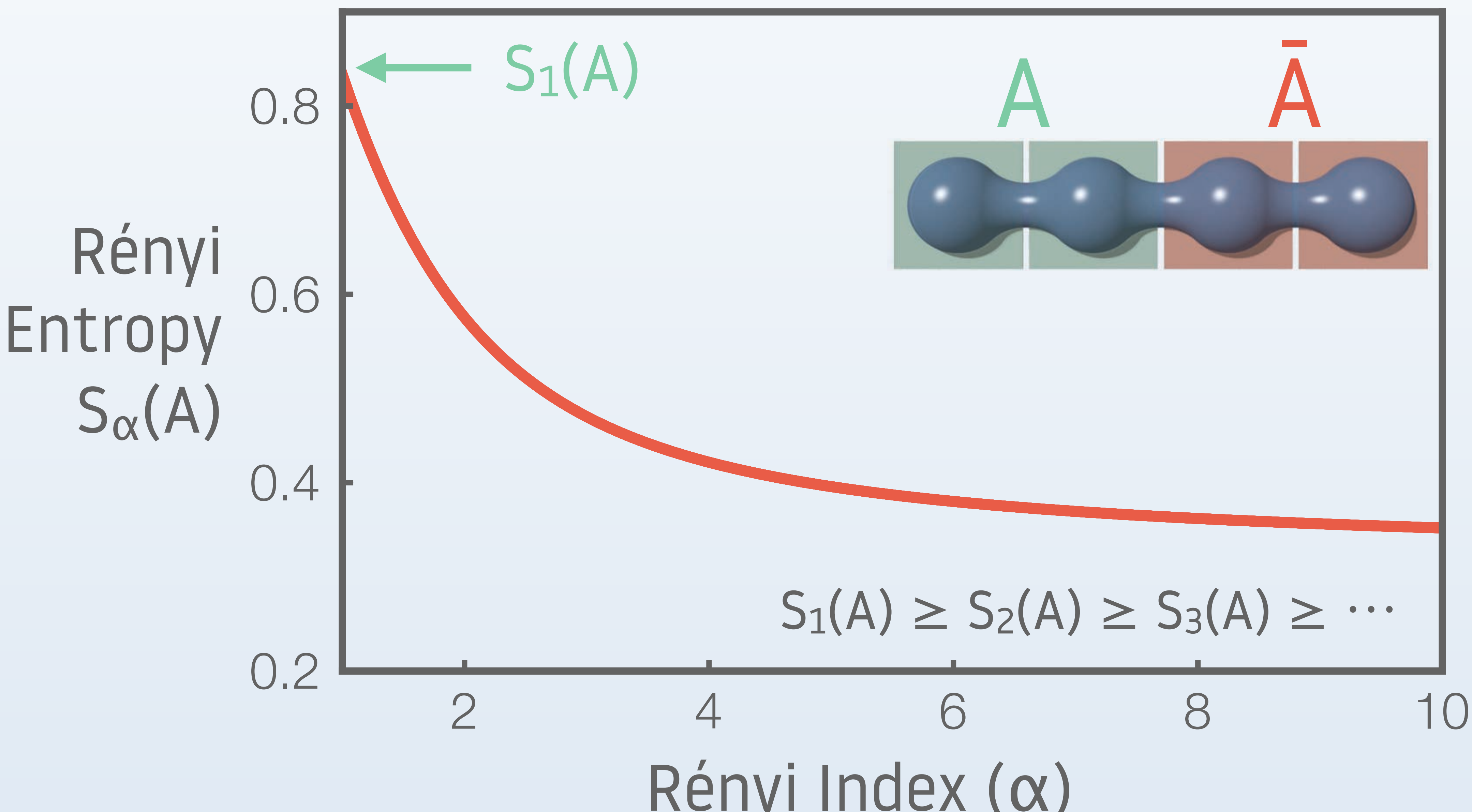
S(A) : how entangled are A and Ā

Rényi Entanglement Entropies

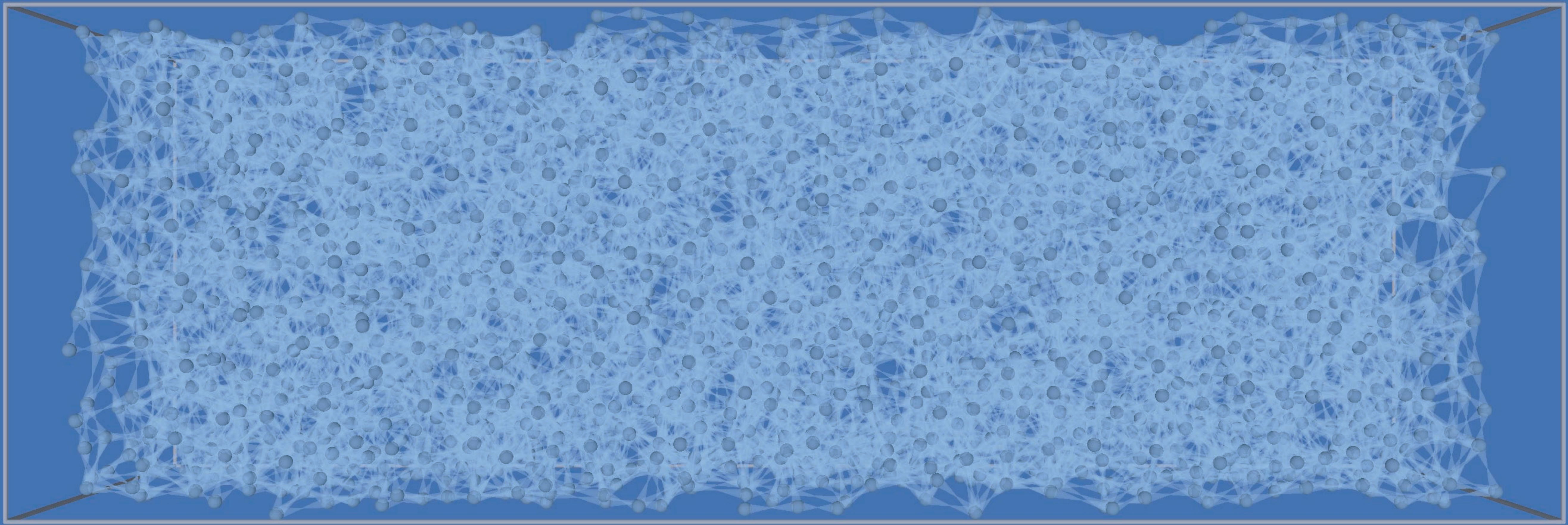
Rényi

An alternate measure of entanglement

$$S_\alpha(A) = \frac{1}{1-\alpha} \log \mathrm{Tr}\, \rho_A^\alpha \;\Rightarrow\; \lim_{\alpha \to 1} S_\alpha(A) = -Tr \rho_A \log \rho_A$$



How does quantum *indistinguishability* affect entanglement?



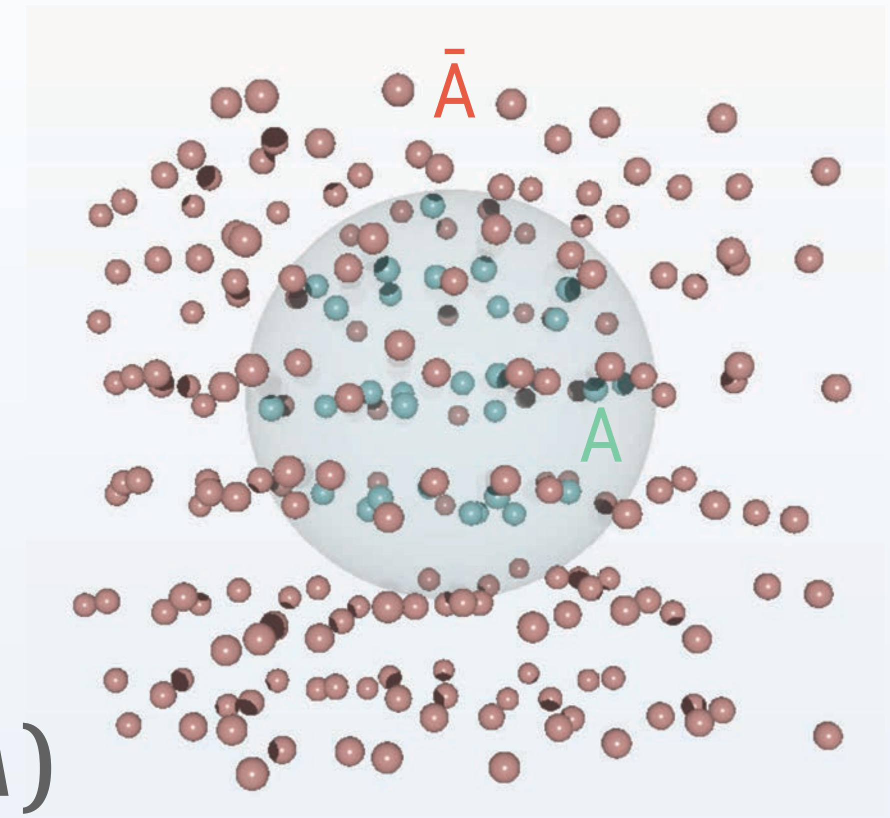
Bipartitions of identical particles

Different ways to partition ground state!

Mode Bipartition

Constructed from the Fock space of single-particle modes

$$|\Psi\rangle = \sum_{\boldsymbol{n}_A \boldsymbol{n}_{\bar{A}}} c_{\boldsymbol{n}_A \boldsymbol{n}_{\bar{A}}} \left|\boldsymbol{n}_A\right\rangle \otimes \left\langle \boldsymbol{n}_{\bar{A}}\right|$$



$$\rho_A \rightarrow S(A)$$

Particle Bipartition

Label a subset of *n* particles

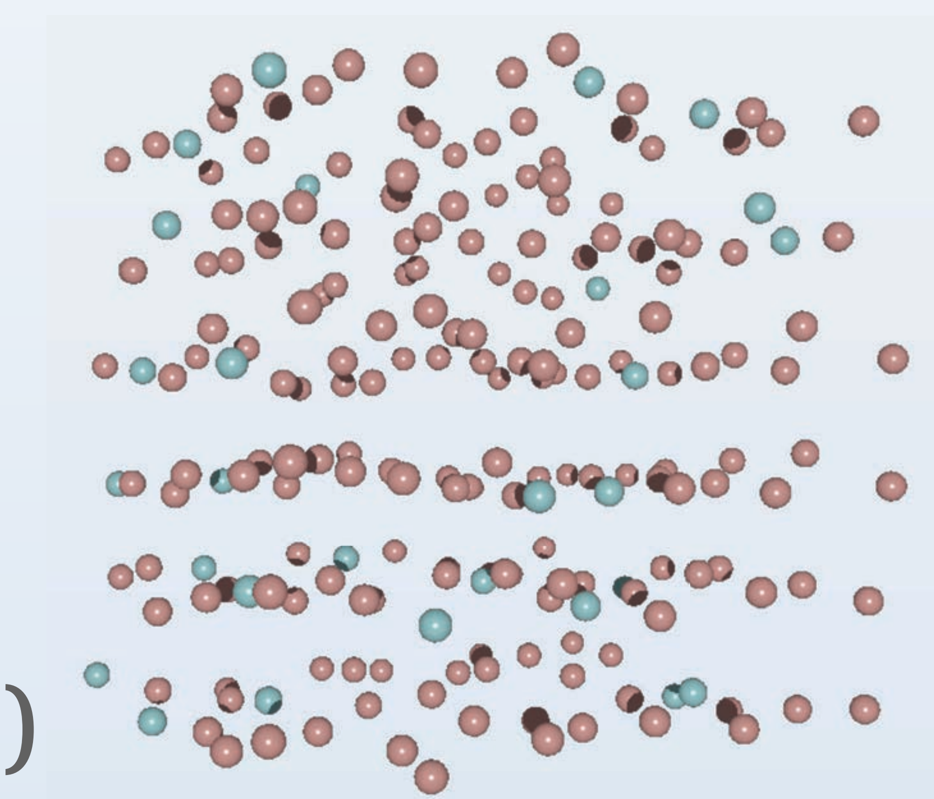
$n_A = n$

$n_B = N - n$

$$|\Psi\rangle = |\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N\rangle$$

$$\rho_n = \int d\boldsymbol{r}_{n+1} \cdots d\boldsymbol{r}_N \langle\Psi|\rho|\Psi\rangle$$

n-body density matrix



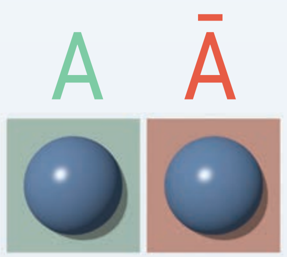
$$\rho_n \rightarrow S(n)$$

Example: a simple quantum liquid I

1d Bose-Hubbard model

$$H = -J\sum_j \left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) + \frac{U}{2}\sum_j n_j(n_j - 1)$$

example: L = 2, N = 2 spatial bipartition:



$$|\Psi\rangle = \alpha\,|20\rangle + \beta\,|11\rangle + \gamma\,|02\rangle$$

$$\rho_A = \mathrm{Tr}_{\bar{A}}\,\rho = \sum_{n=0}^{2} {}_{\bar{A}}\langle n|\Psi\rangle\,\langle\Psi|n\rangle_{\bar{A}} = \begin{pmatrix} |\alpha|^2 & 0 & 0 \\ 0 & |\beta|^2 & 0 \\ 0 & 0 & |\gamma|^2 \end{pmatrix}$$

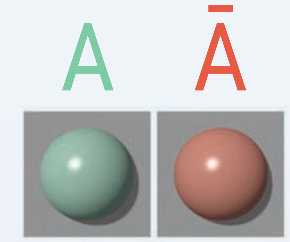
$$S_1(\rho_A) = -\mathrm{Tr}\,\rho_A \ln \rho_A = -|\alpha|^2 \ln|\alpha|^2 - |\beta|^2 \ln|\beta|^2 - |\gamma|^2 \ln|\gamma|^2$$

Example: a simple quantum liquid II

1d Bose-Hubbard model

$$H = -J\sum_j \left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) + \frac{U}{2}\sum_j n_j(n_j - 1)$$

example: L = 2, N = 2 particle bipartition:



$$|\Psi\rangle = \alpha|1_1 2_1\rangle + \frac{\beta}{\sqrt{2}}\left(|1_1 2_2\rangle + |1_2 2_1\rangle\right) + \gamma|1_2 2_2\rangle$$

$$\rho_1 = \sum_{i=1}^{2} \langle 2_i|\Psi\rangle\langle\Psi|2_i\rangle = \begin{pmatrix} \alpha^2 + \frac{\beta^2}{2} & \frac{\alpha^*\beta+\beta^*\gamma}{\sqrt{2}} \\ \frac{\alpha\beta^*+\beta\gamma^*}{\sqrt{2}} & \gamma^2 + \frac{\beta^2}{2} \end{pmatrix}$$

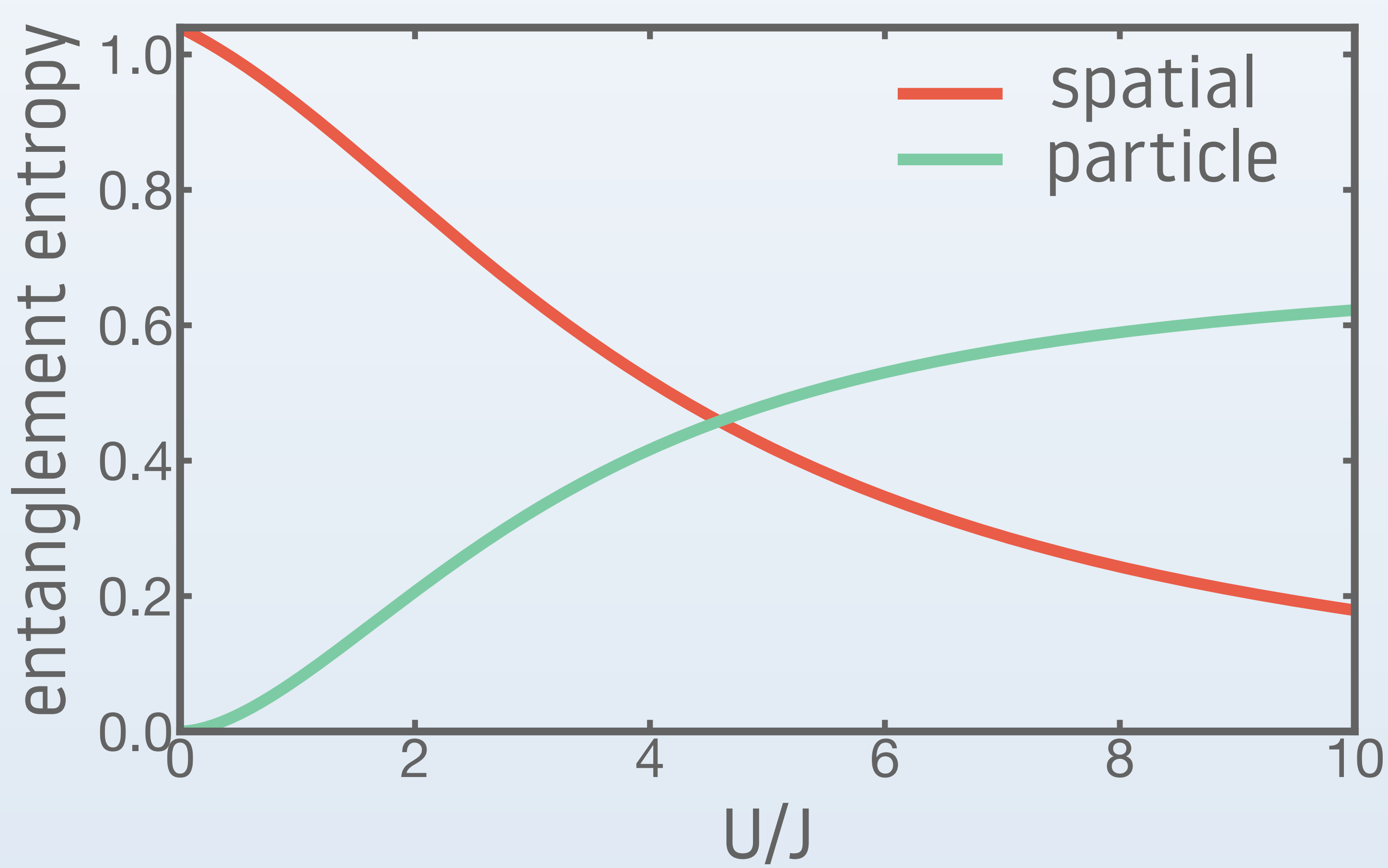
Example: a simple quantum liquid III

1d Bose-Hubbard model

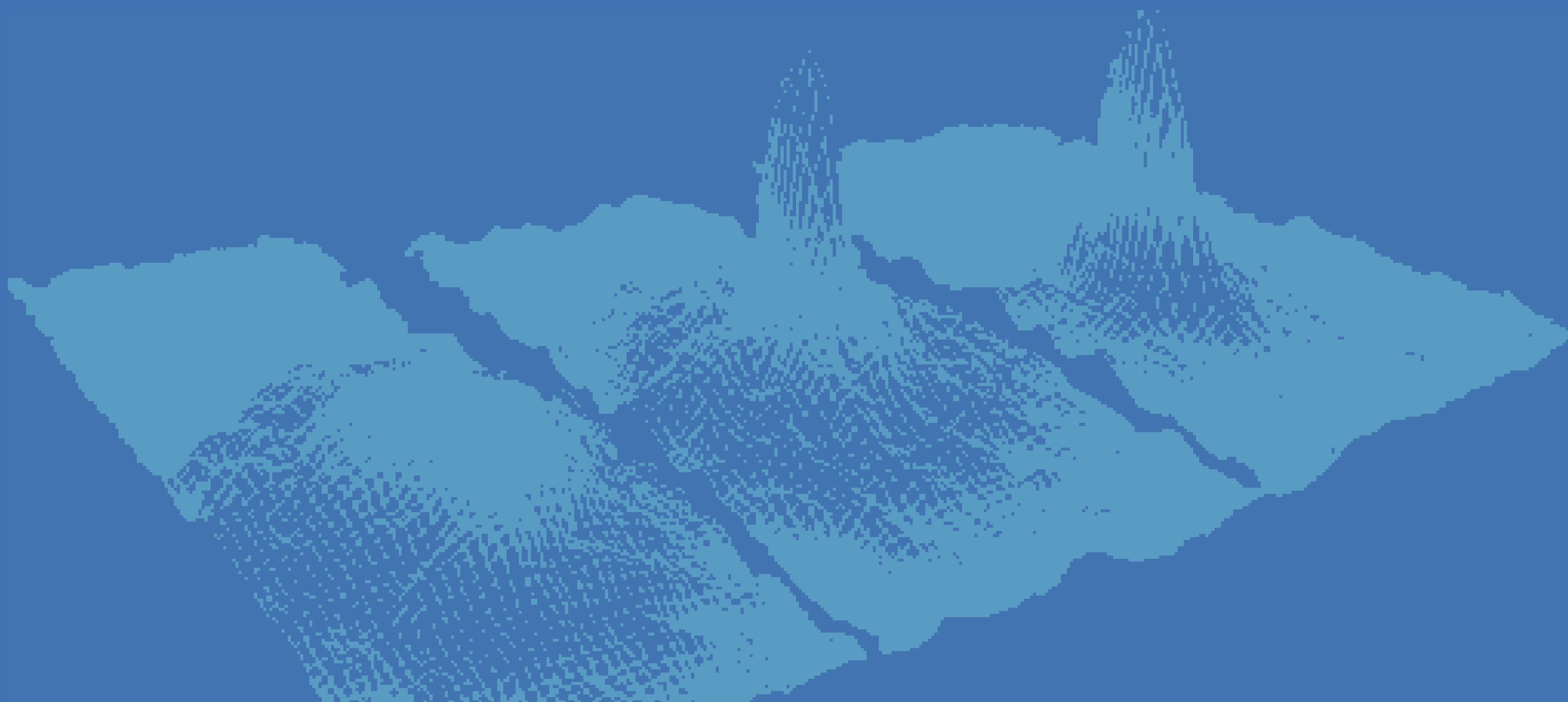
$$H = -J\sum_j \left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) + \frac{U}{2}\sum_j n_j\left(n_j - 1\right)$$

example: L = 2, N = 2

different bipartitions can provide complimentary information on phases, interactions & statistics



Can we use the *entanglement* in quantum fluids as a *resource* for information processing?



Or is it all just *fluffy bunnies?*

J. Dunningham, A. Rau, and K. Burnett, Science 307, 872 (2005)

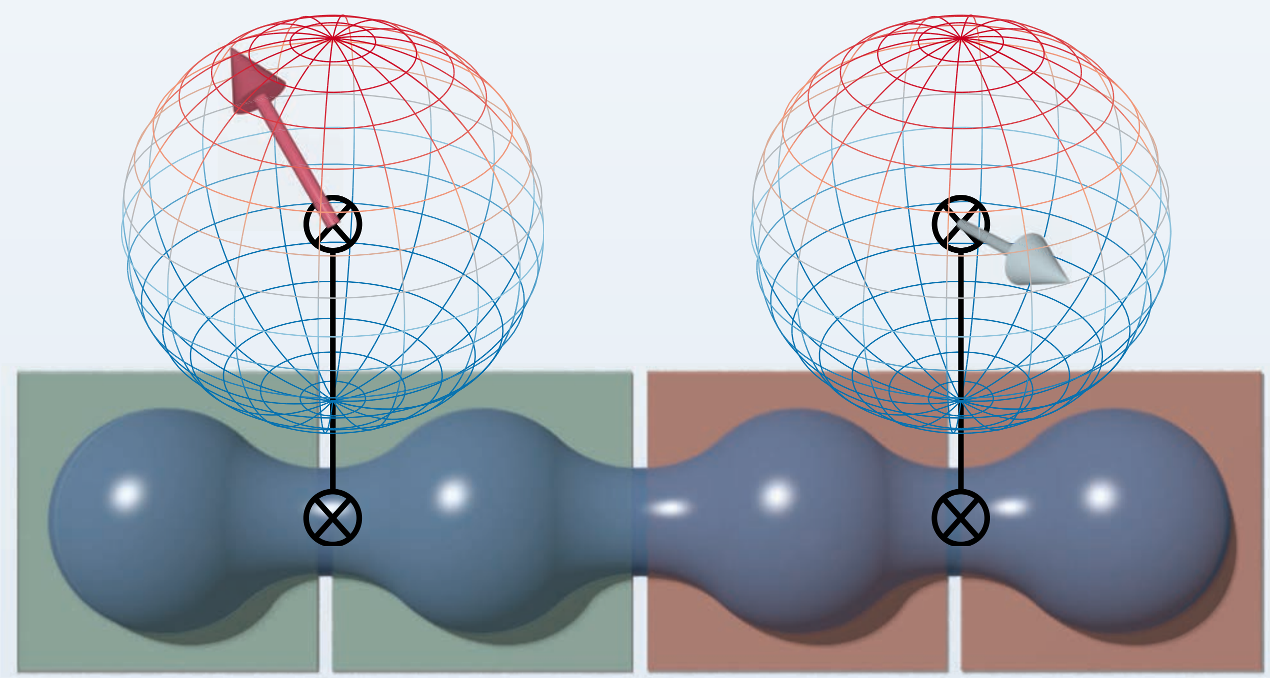
Using entanglement as a resource requires ability to perform local physical operations on subsystems

Particle Entanglement

inaccessible due to the indistinguishability of particles

N. Killoran, M. Cramer, and M. B. Plenio, PRL 112, 150501 (2014)

$N = 2,\ L = 4$



$x_1 = ?,\ x_2 = ?$

Spatial Entanglement

particle number conservation prohibits swapping all entanglement to register

H. M. Wiseman and J. A. Vaccaro, PRL 91, 097902 (2003)

$$\widehat{SWAP}\left[\left(|1\rangle_A \otimes |1\rangle_B + |2\rangle_A \otimes |0\rangle_B + |0\rangle_A \otimes |2\rangle_B\right) \otimes |0\rangle_{reg}\right]$$

Operational entanglement

Get around these difficulties by combining the two measures.

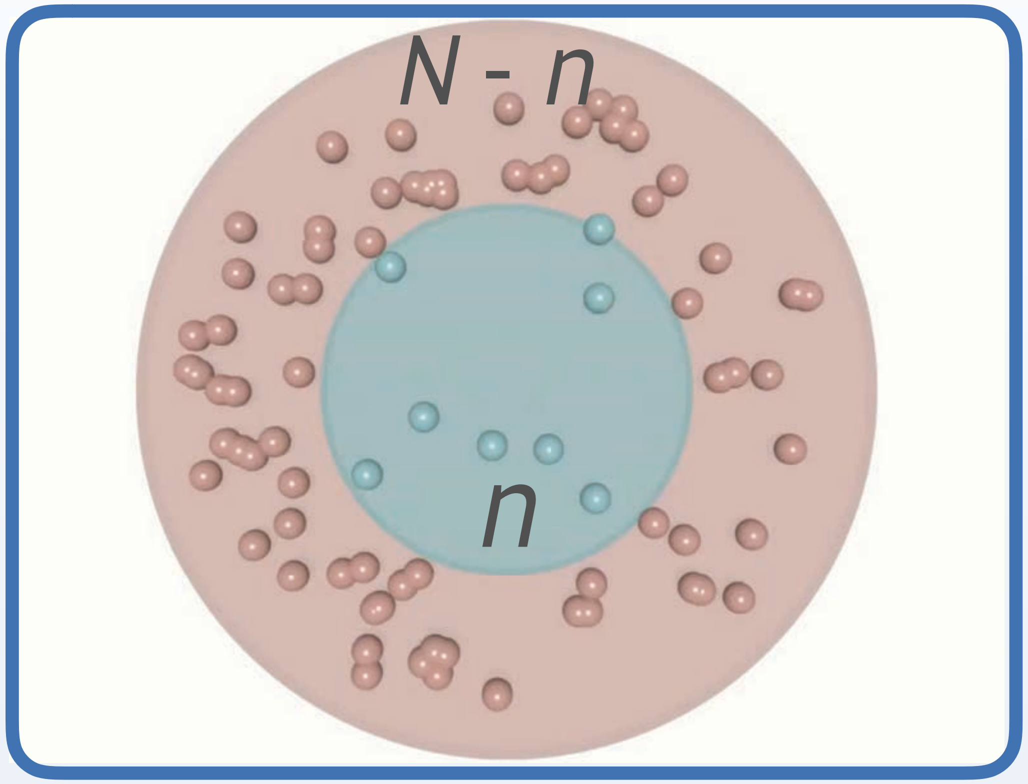
H. M. Wiseman and J. A. Vaccaro, PRL 91, 097902 (2003)

$$S^{\mathrm{op}}(A) = \sum_n P_n S(A_n)$$

$$\rho_{A_n} = \frac{1}{P_n}\hat{P}_n \rho_A \hat{P}_n$$

probability

projection operator



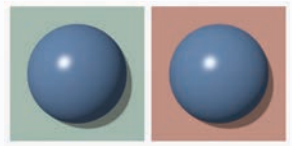
$$S^{\mathrm{op}}(A) < S(A)$$

$$S^{\mathrm{op}}(A) > 0 \Rightarrow S(n) > 0$$

Maximal amount of entanglement that can be produced between quantum registers by local operations.

Operational Entanglement Example

example: L = 2 $|\Psi\rangle = \alpha\,|20\rangle + \beta\,|11\rangle + \gamma\,|02\rangle$



$$\rho = |\Psi\rangle\langle\Psi| = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \beta\alpha^* & |\beta|^2 & \beta\gamma^* \\ \gamma\alpha^* & \gamma\beta^* & |\gamma|^2 \end{pmatrix} \qquad \rho_A = \mathrm{Tr}_{\bar{A}}\,\rho = \begin{pmatrix} |\alpha|^2 & 0 & 0 \\ 0 & |\beta|^2 & 0 \\ 0 & 0 & |\gamma|^2 \end{pmatrix}$$

$$S_1^{\mathrm{op}}(A) = \sum_n P_n S_1(A_n) \qquad \rho_{A_n} = \frac{1}{P_n}\hat{P}_n \rho_A \hat{P}_n$$

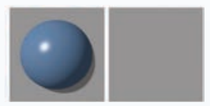
$$\rho_{A_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \rho_{A_1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \rho_{A_0} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

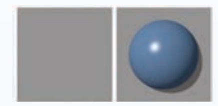
$$S_1^{\mathrm{op}}(A) = 0$$

need at least 2 states in the subsystem

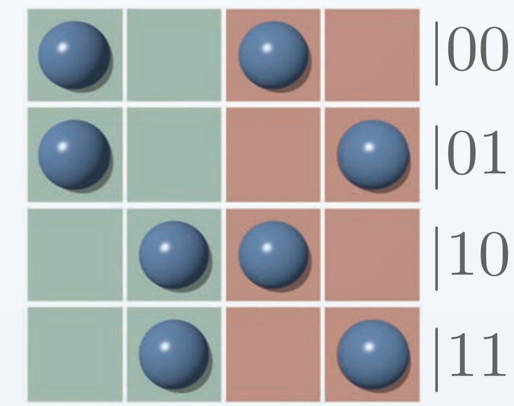
Experimental Measurement

Density matrix is generally inaccessible

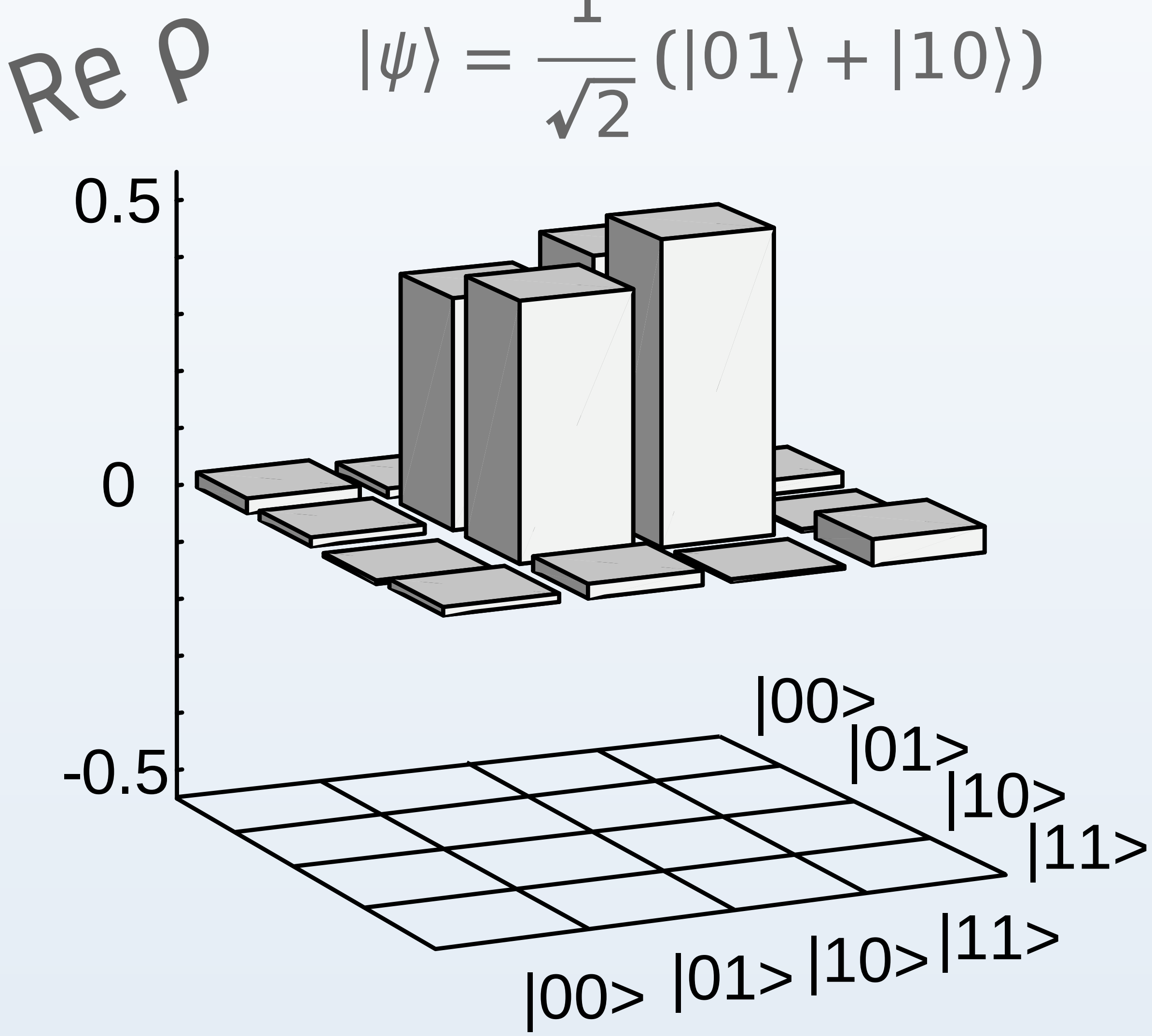

$= |0\rangle$

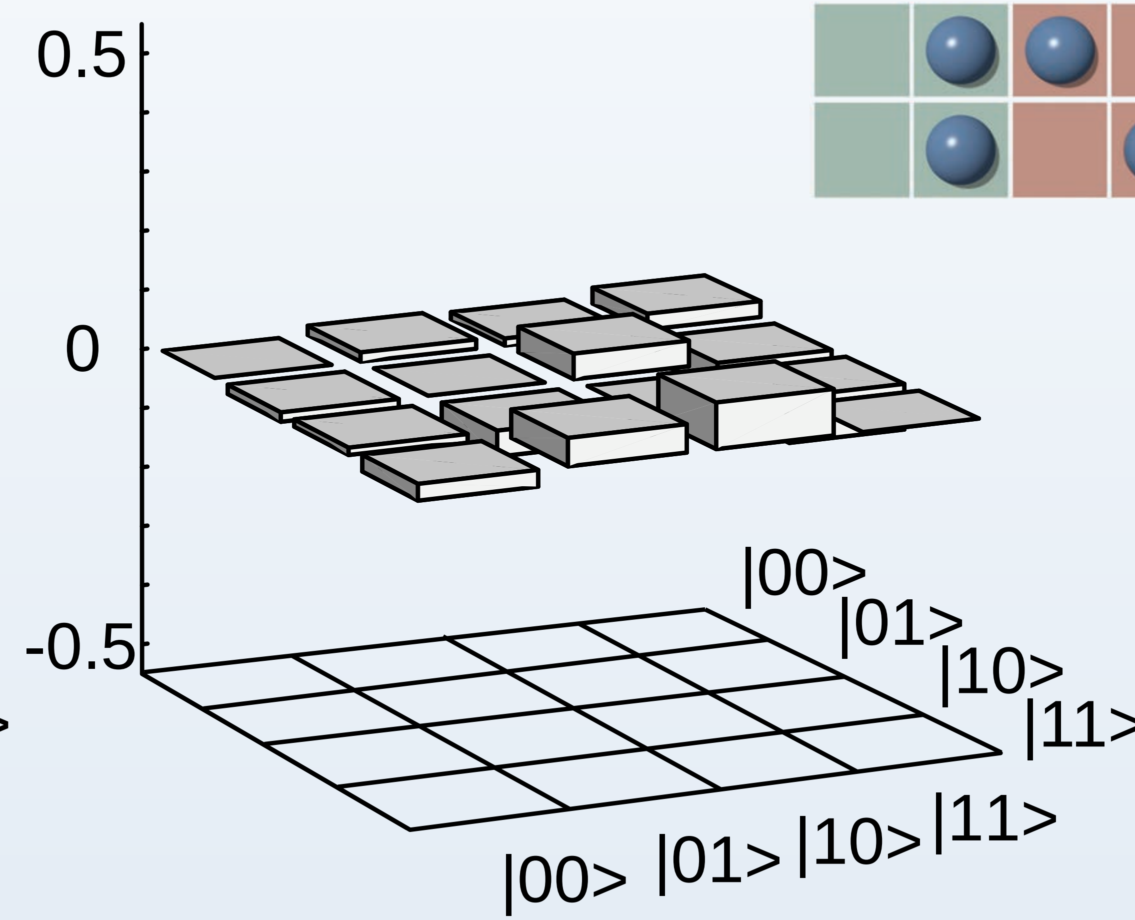

$= |1\rangle$

C. F. Roos et al, PRL 92, 220402 (2004)



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$



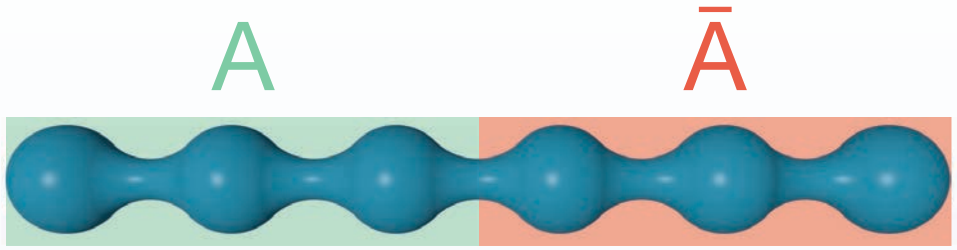


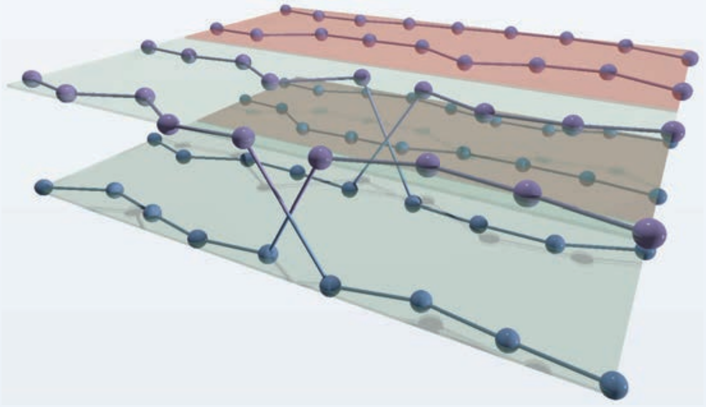
Measurement becomes exponentially difficult!

4 particles on 4 sites: $\rho \sim 10^5$ entries

Entanglement and Entropy

quantifying uncertainty in many-body systems



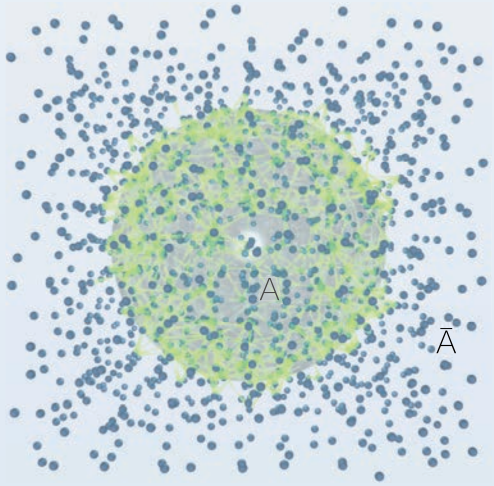


Measuring Entanglement

SWAP algorithm in experiment and quantum Monte Carlo

Results for Quantum Liquids & Gases

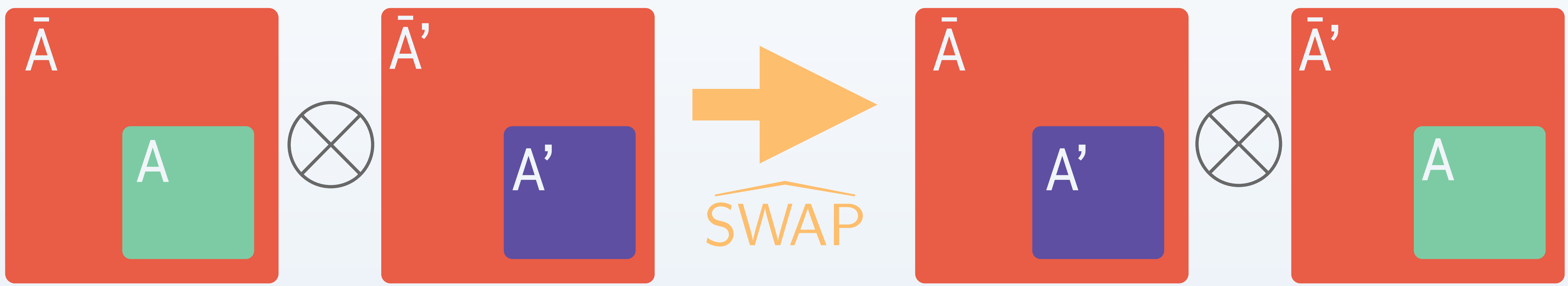
benchmarking, scaling and the area law



The Replica Method

Computing Rényi entanglement entropies by swapping subregions between non-interacting identical copies

$\alpha = 2 \rightarrow 2$ replicas of system — swap subregions



claim: $S_2(A) = -\log \mathrm{Tr}\,\rho_A^2 = -\log \left\langle \widehat{\mathrm{SWAP}} \right\rangle$

$$Tr\,\rho_A \rho_{A'} = \sum_{a,a'} \langle a|\rho_A|a'\rangle \langle a'|\rho_{A'}|a\rangle \longleftarrow \rho_A = \sum_{b\in\bar{A}} \langle b|\rho|b\rangle$$

$$= \sum_{a,a'} \sum_{b,b'} \langle ab|\rho|a'b\rangle \langle a'b'|\rho'|ab'\rangle$$

$$= \sum_{a,a'} \sum_{b,b'} \langle aba'b'|\rho \otimes \rho'|a'bab'\rangle$$

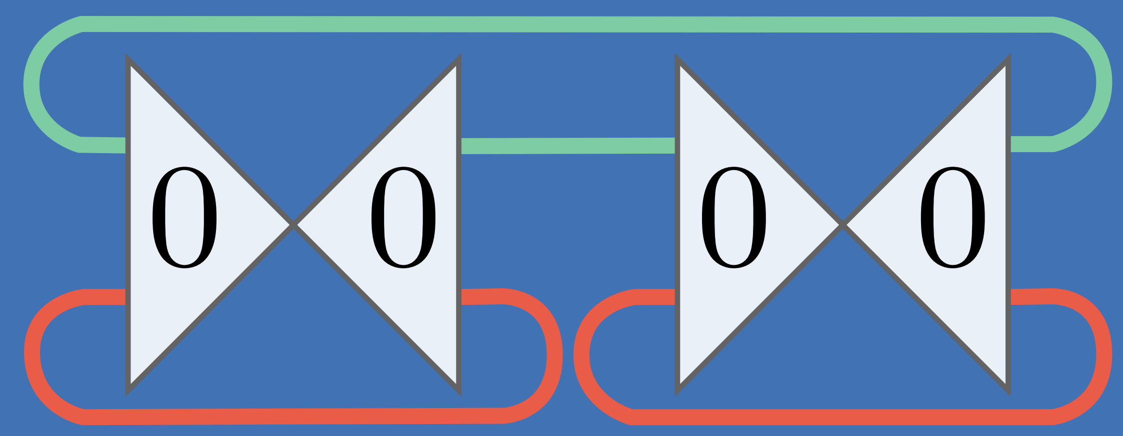
$$= \sum_{a,a'} \sum_{b,b'} \langle aba'b'|\rho \otimes \rho' \widehat{SWAP}|aba'b'\rangle = Tr\left[(\rho \otimes \rho')\widehat{SWAP}\right]$$

expectation value in replicated ensemble $\langle \widehat{SWAP} \rangle_{\rho\otimes\rho'}$

P. Calabrese and J. Cardy, J. Stat. Mech.: Theor. Exp. P06002 (2004)

Rényi entropies can be computed via local expectation values!

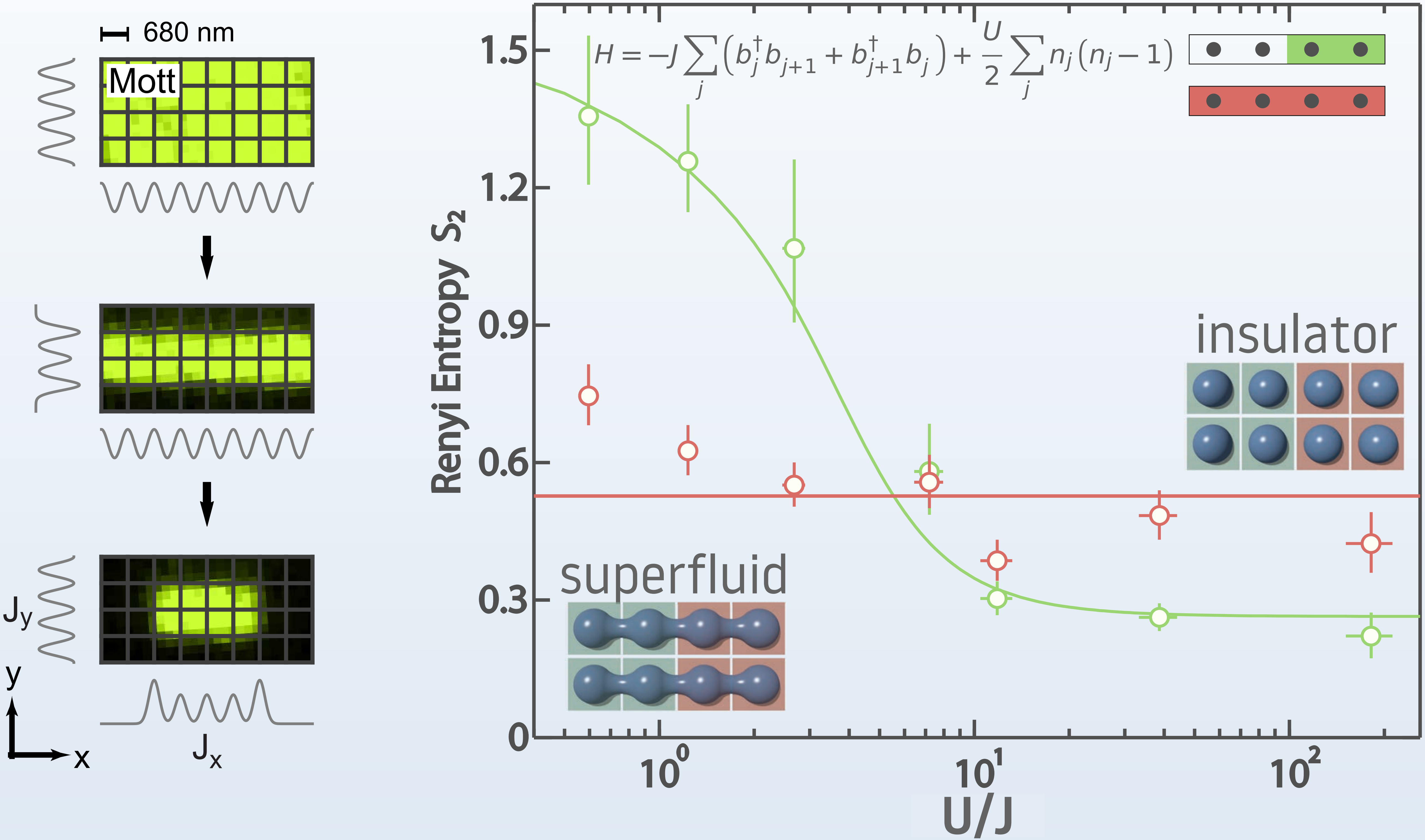
$\mathrm{Tr}\,\rho_A^2 \Leftrightarrow$



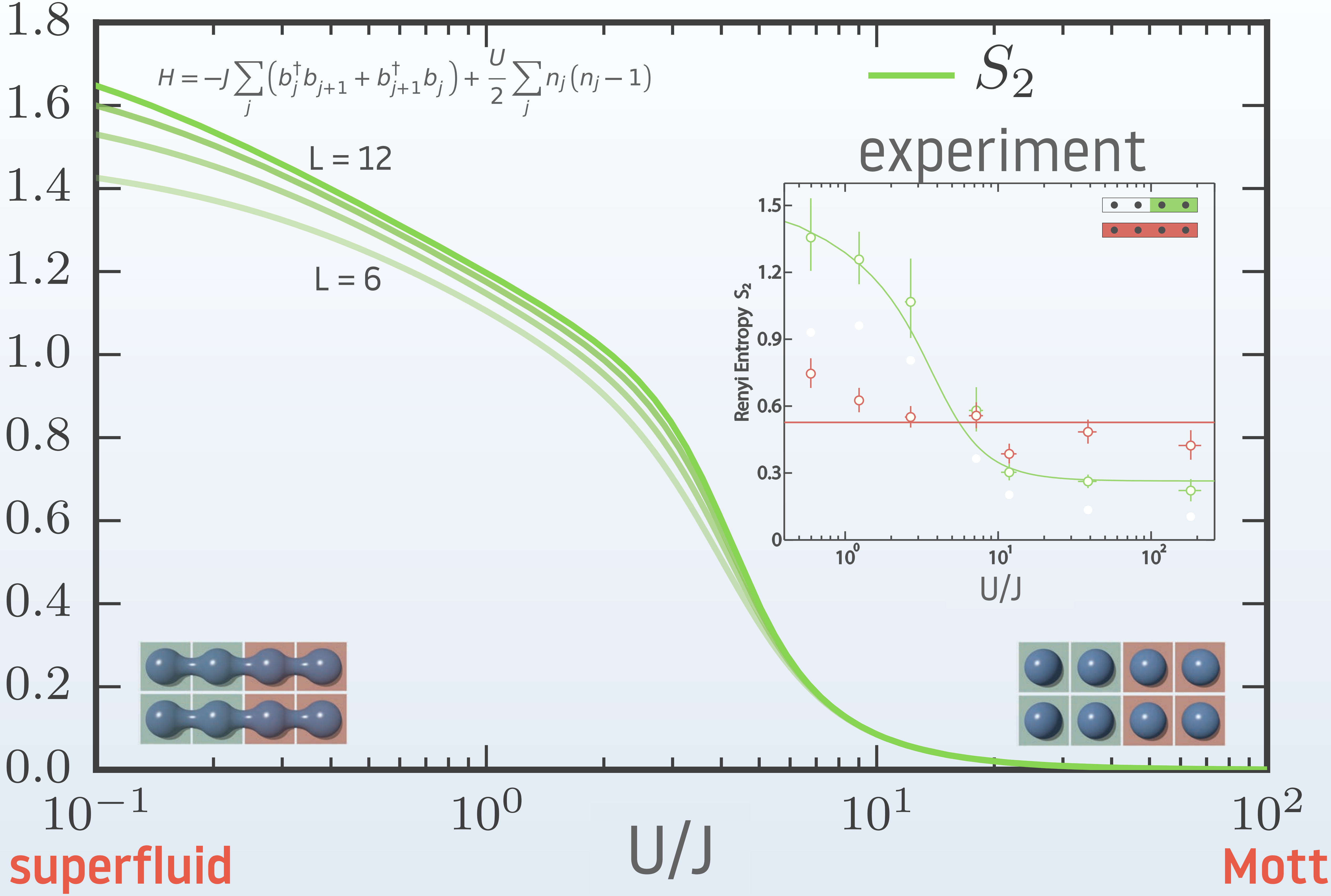
Experimental Measurement

C. Hong, Z. Ou, and L. Mandel, PRL 59 2044, (1987)
A. J. Daley, H. Pichler, J. Schachenmayer, and P. Zoller, Phys. Rev. Lett. 109, 020505 (2012)
R. Islam, R. Ma, P. M. Preiss, M. E. Tai, A. Lukin, M. Rispoli, and M. Greiner, Nature 528, 77 (2015)

IDEA: Use bosonic Hong-Ou-Mandel interference with atoms

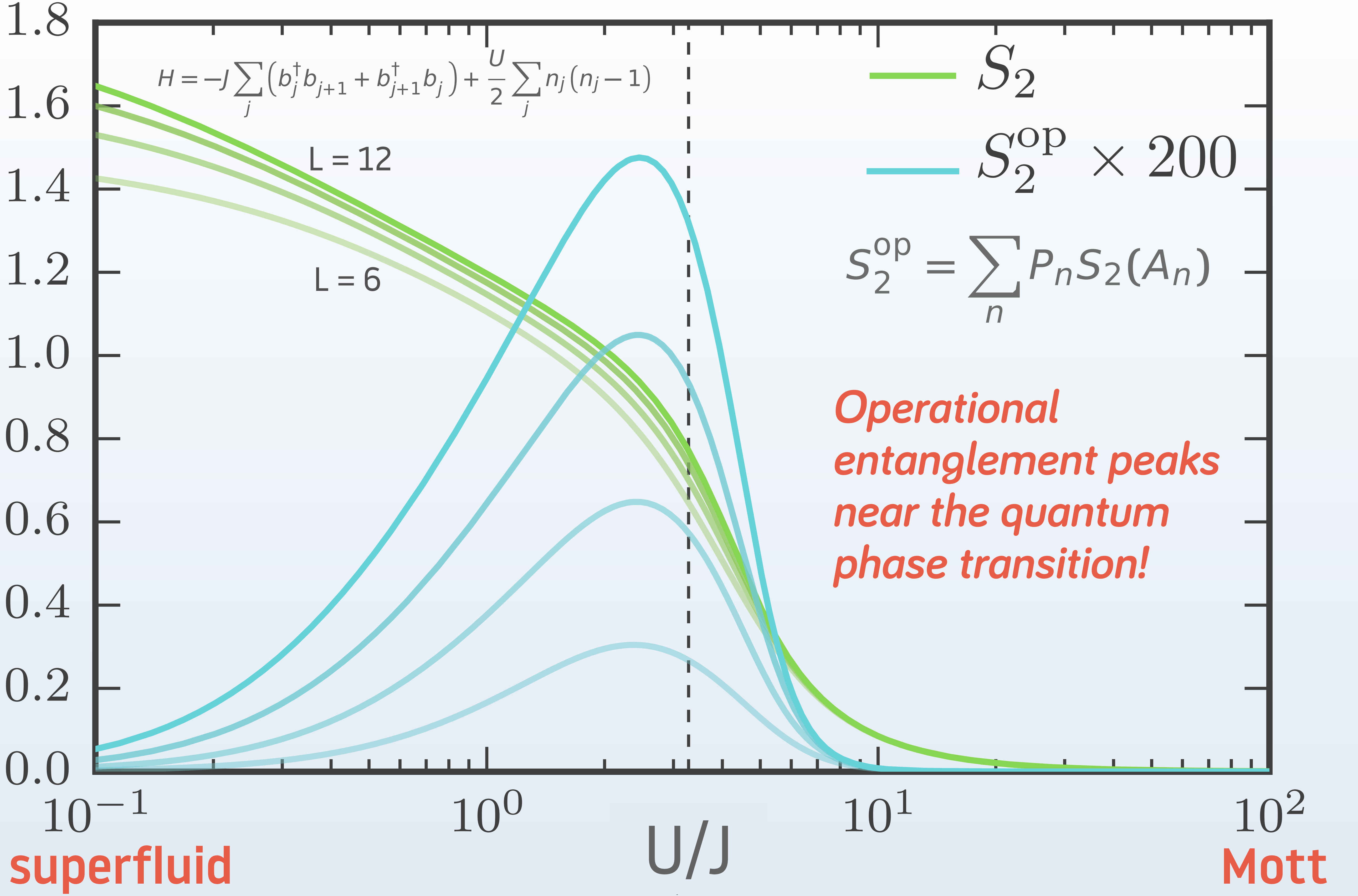


Exact Diagonalization Results



R. Melko, C. Herdman, D. Iouchtchenko, P.-N. Roy and A.D. Phys. Rev. A, 93, 042336 (2016)

Exact Diagonalization Results



R. Melko, C. Herdman, D. Iouchtchenko, P.-N. Roy and A.D. Phys. Rev. A, 93, 042336 (2016)

Exact diagonalization is *limited* to *small* systems with *discrete* Hilbert spaces

Can we get **bigger?**

Path Integral Ground State QMC

Description

$$\hat{H} = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \hat{\nabla}_i^2 + \sum_{i=1}^{N} \hat{\mathcal{V}}_i + \sum_{i<j} \hat{\mathcal{U}}_{ij}$$

N interacting particles in d-dimensions

Configurations

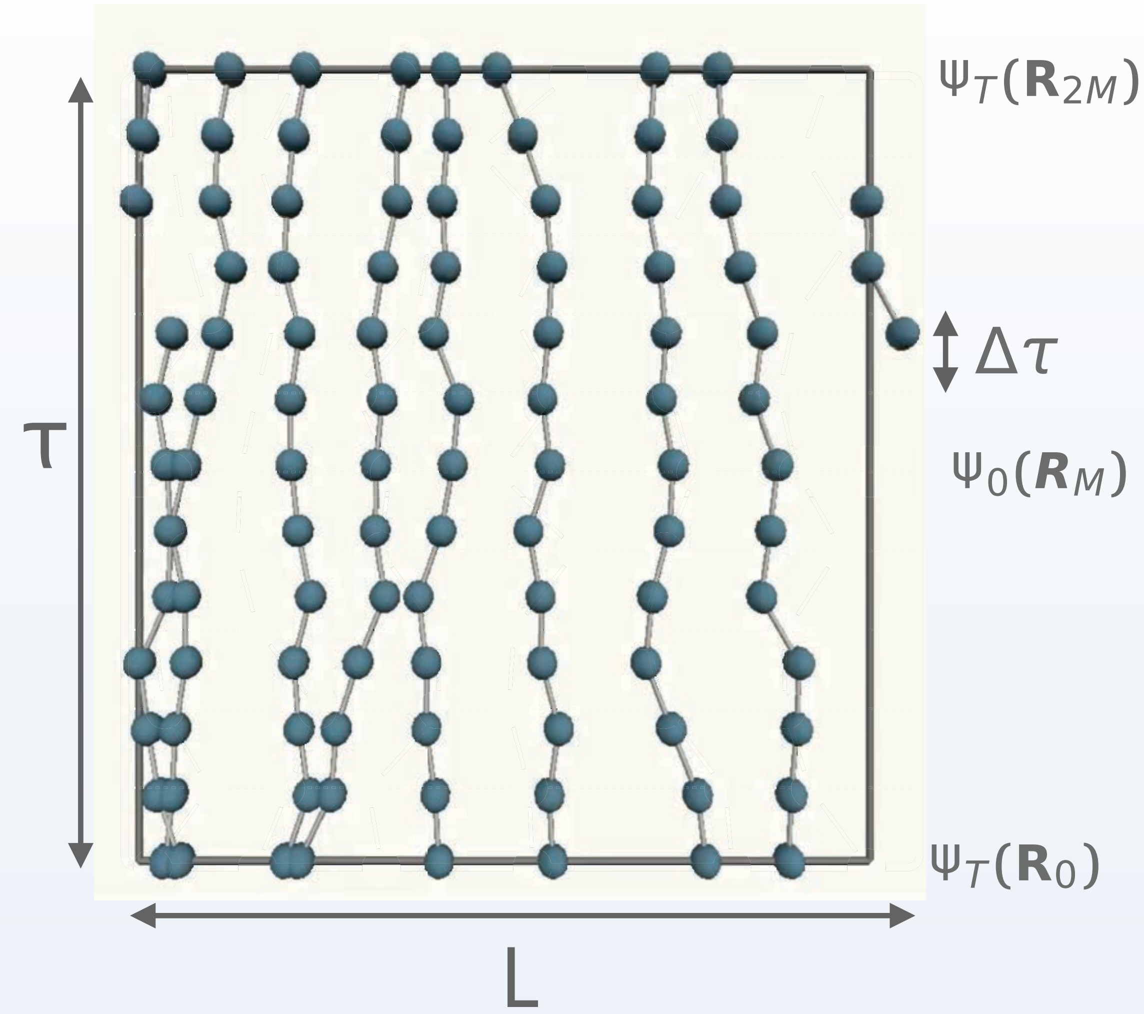
projecting a trial wavefunction to the ground state $|\Psi_0\rangle = \lim_{\tau \to \infty} e^{-\tau \hat{H}} |\Psi_T\rangle$

gives discrete imaginary time worldlines constructed from products of the short time propagator $G(\boldsymbol{R}, \boldsymbol{R}'; \Delta\tau) = \langle \boldsymbol{R} | e^{-\Delta\tau \hat{H}} | \boldsymbol{R}' \rangle$

Observables

exact method for computing ground state expectation values

$$O_\tau = \frac{\langle \Psi_T | e^{-\tau \hat{H}} \hat{O} e^{-\tau \hat{H}} | \Psi_T \rangle}{\langle \Psi_T | e^{-2\tau \hat{H}} | \Psi_T \rangle}$$

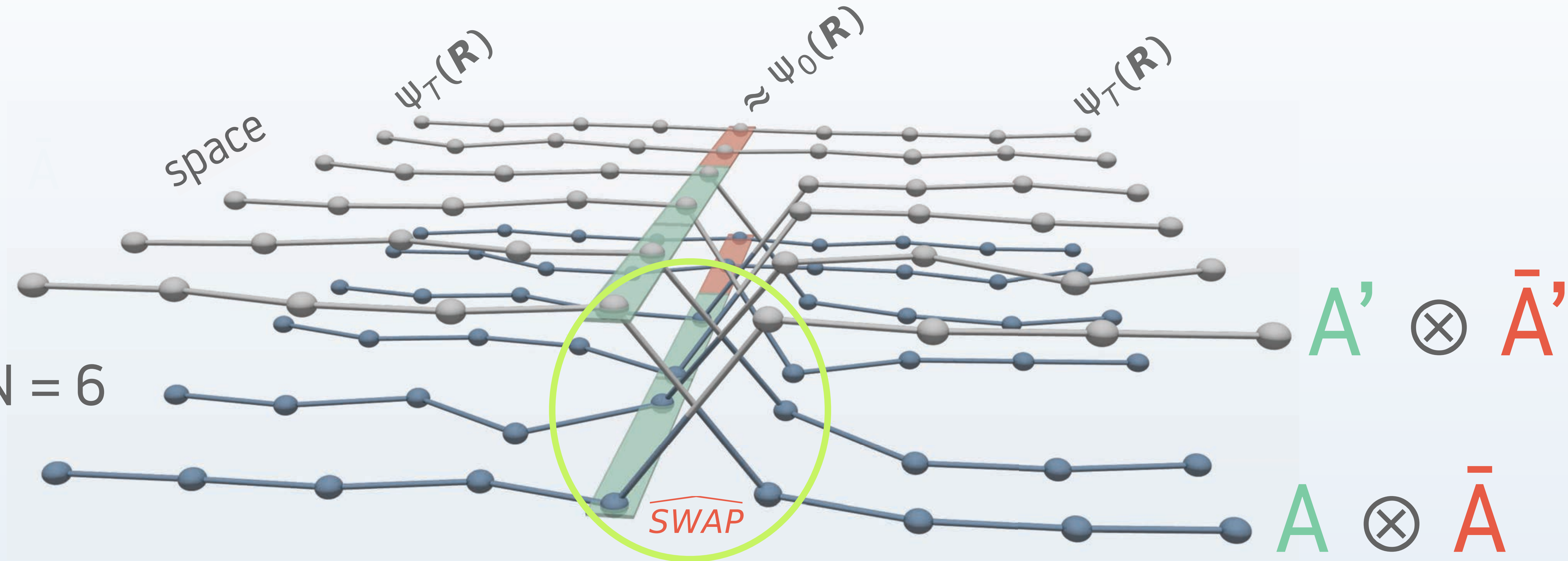


Updates

Local and non-local bead updates with weights given by $\pi(\mathbf{X})$

Porting the Replica Method to PIGS

Break paths at the center time slice τ, measure $\widehat{SWAP}$ when replicas are linked via short time propagator G.



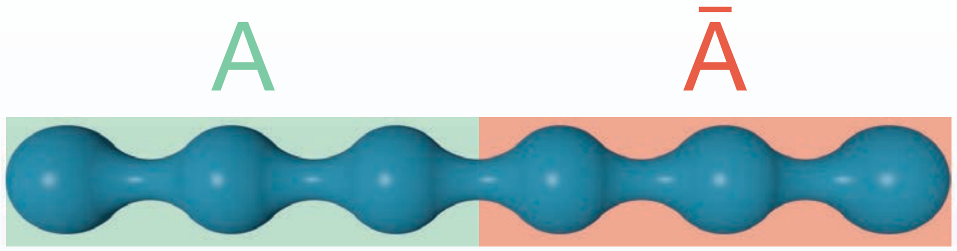
$$\left\langle \widehat{SWAP} \right\rangle = \left\langle G\left(\boldsymbol{R}_{\tau} \otimes \boldsymbol{R}'_{\tau'}, \widehat{SWAP}\left[\boldsymbol{R}_{\tau+\Delta\tau} \otimes \boldsymbol{R}'_{\tau+\Delta\tau}\right]; \Delta\tau\right)\right\rangle$$

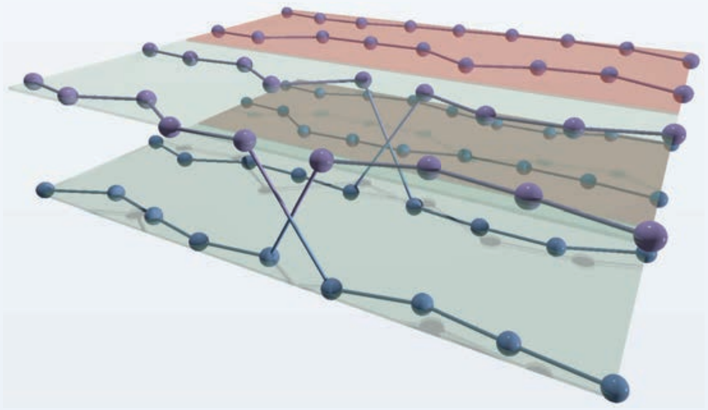
Technology adapted from other QMC flavors

M. B. Hastings, I. González, A. B. Kallin, and R. G. Melko, PRL 104, 157201 (2010)
R. Melko, A. Kallin, and M. Hastings, PRB 82, 100409 (2010)
C. Herdman, R. Melko and A.D. Phys. Rev. B, 89, 140501 (2014)
C. M. Herdman, S. Inglis, P. N. Roy, R. G. Melko, and A.D., PRE 90, 013308 (2014)
T. Grover, Phys. Rev. Lett. 111, 130402 (2013)
Assaad, Lang, Toldin, Phys. Rev. B 89, 125121 (2014)
Broecker and Trebst, J. Stat. Mech. (2014) P08015
J. E. Drut and W. J. Porter, PRB 92, 125126 (2015)

Entanglement and Entropy

quantifying uncertainty in many-body systems



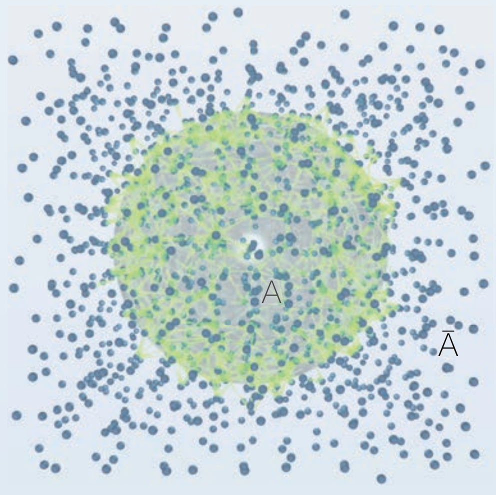


Measuring Entanglement

SWAP algorithm in experiment and quantum Monte Carlo

Results for Quantum Liquids & Gases

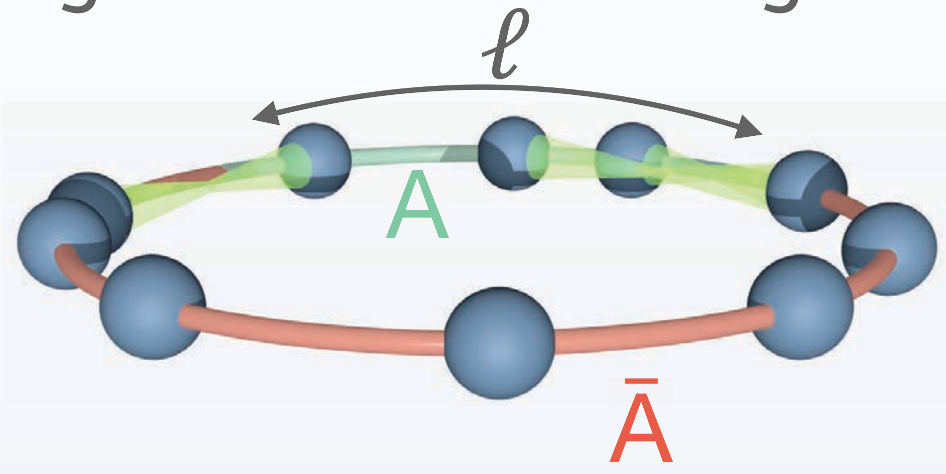
benchmarking, scaling and the area law

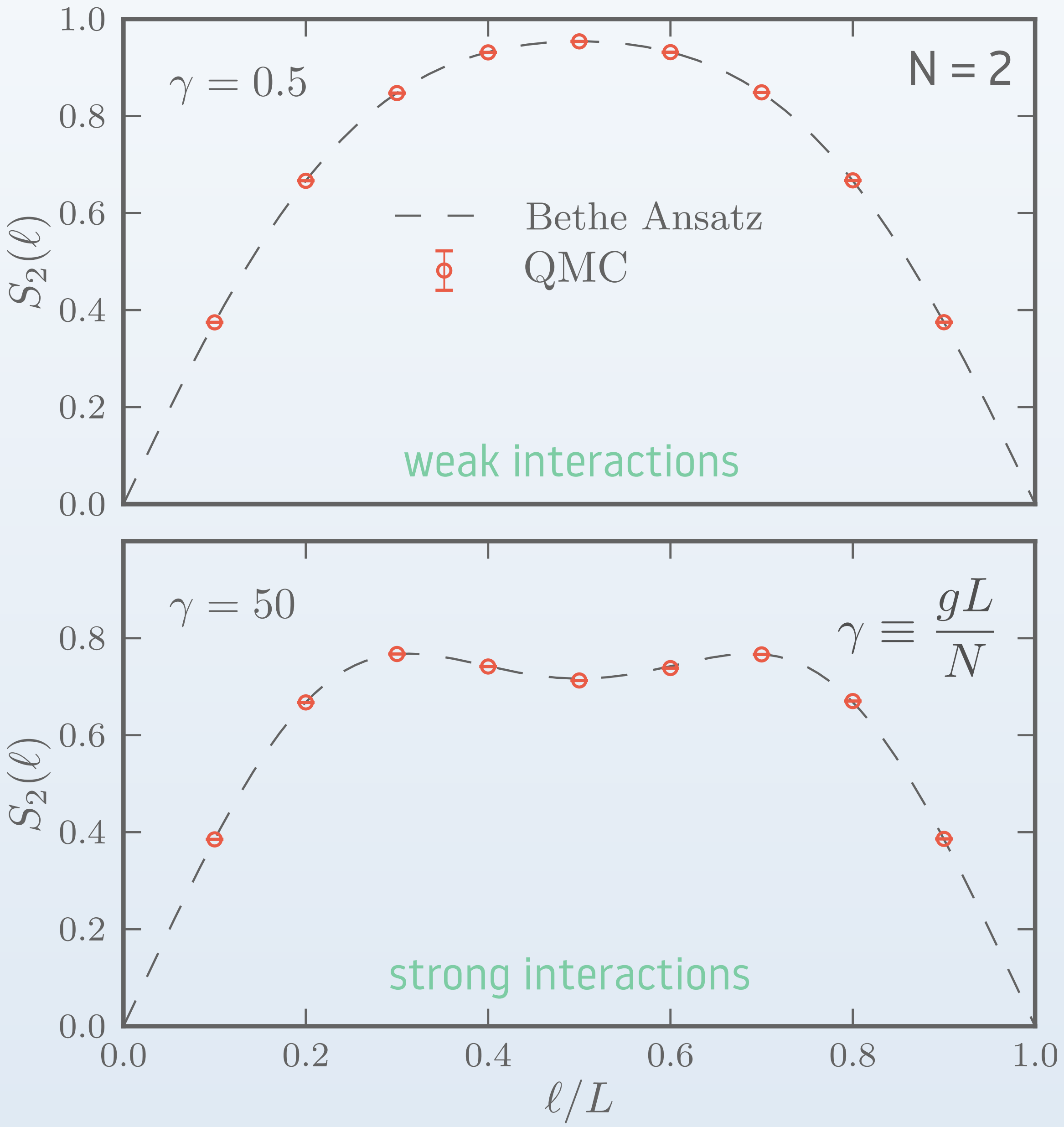


Benchmarking on a Solvable Model

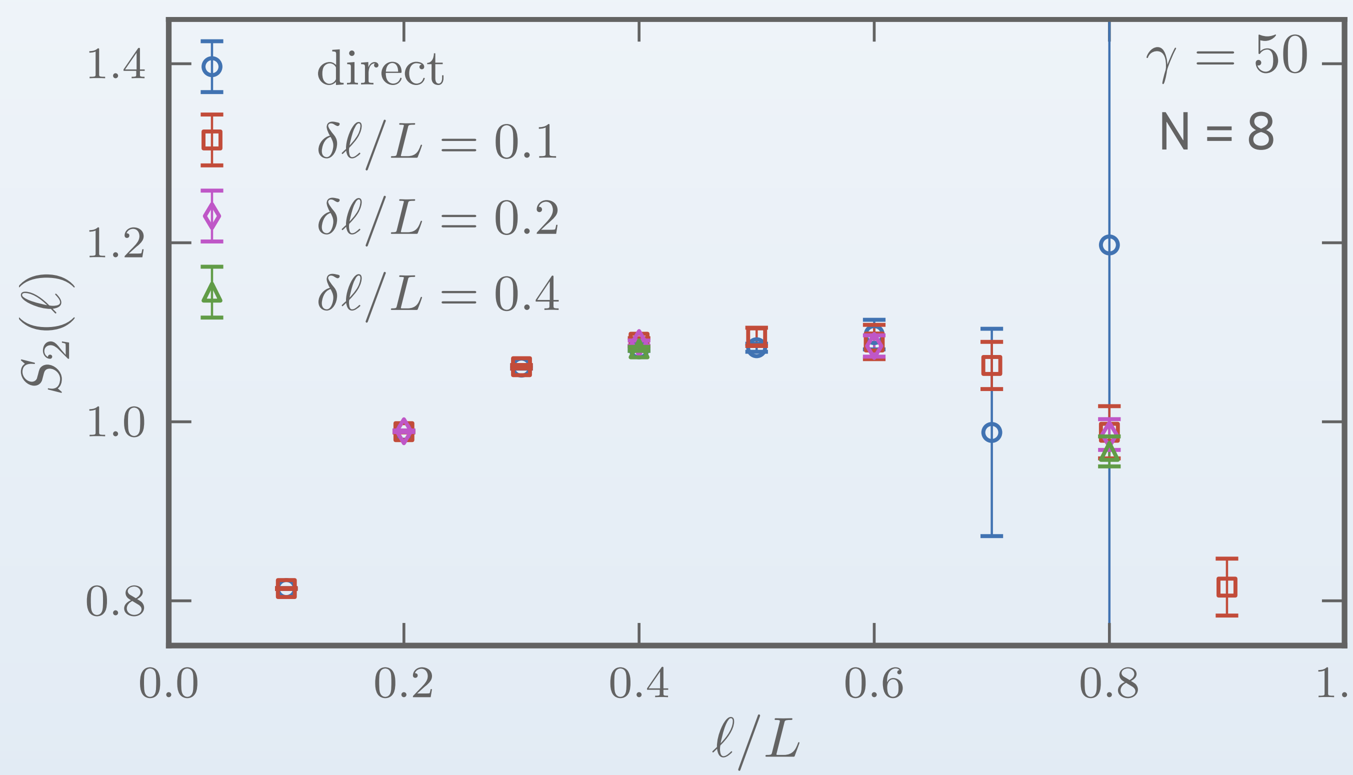
Lieb-Liniger model of δ-function interacting bosons on a ring

$$H = -\frac{1}{2}\sum_{I=1}^{N}\frac{d^2}{dx^2} + g\sum_{i<j}\delta(x_i - x_j)$$





ratio trick

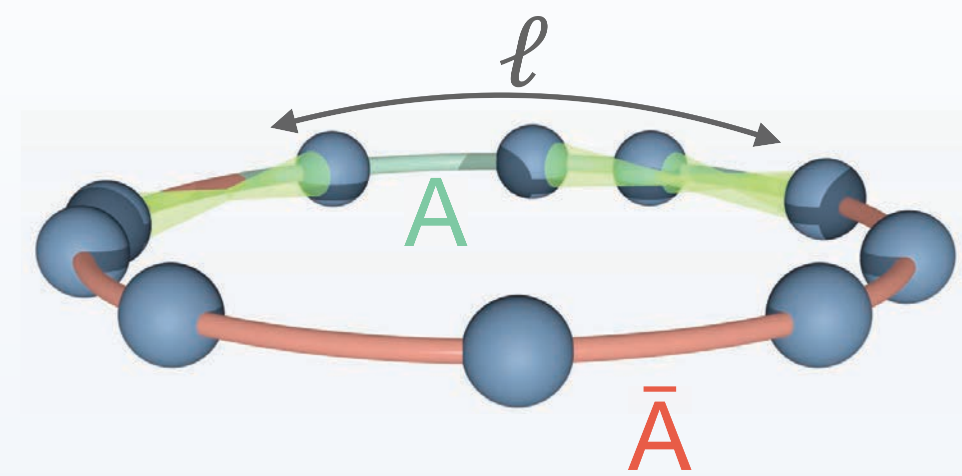


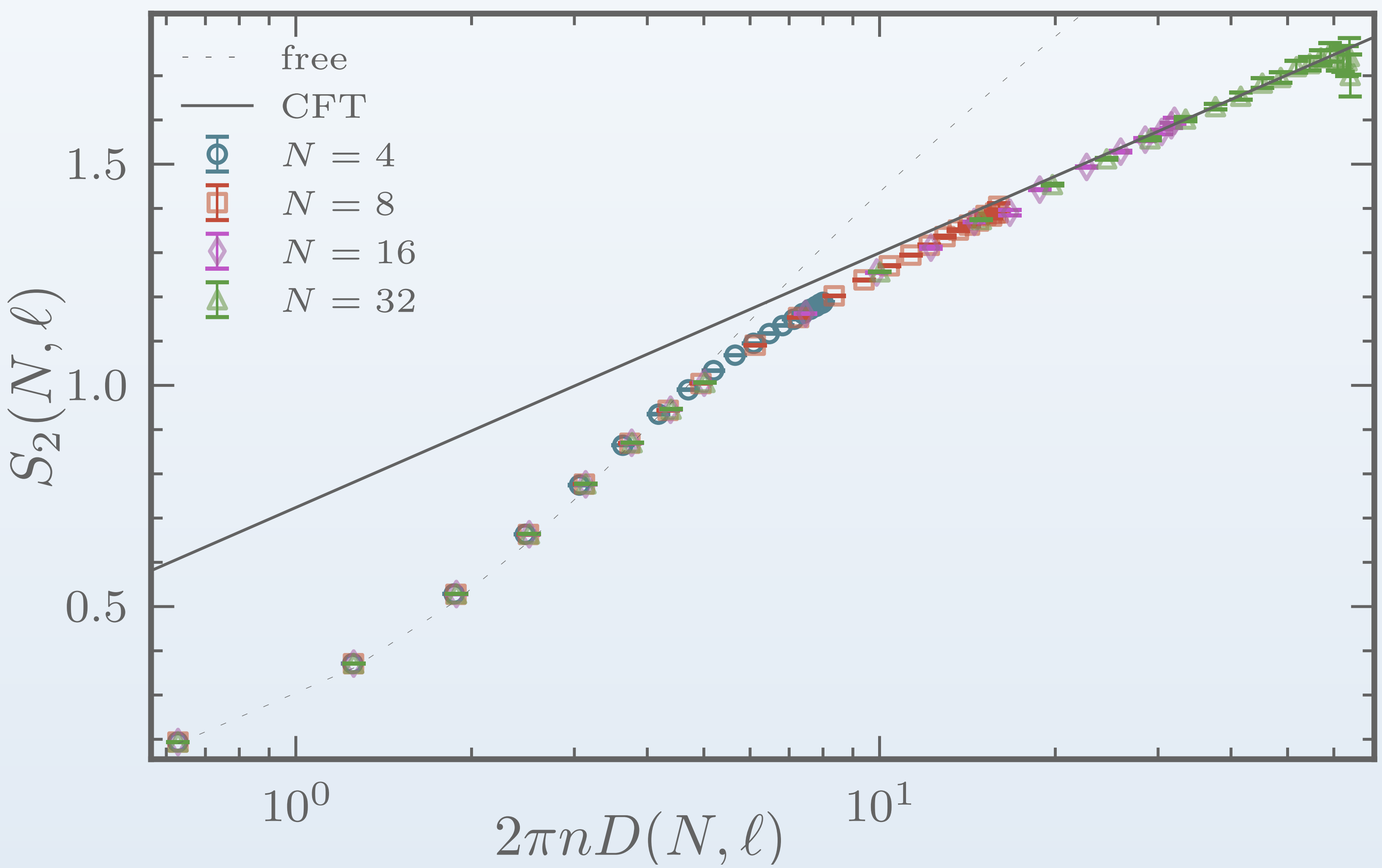
E. H. Lieb and W. Liniger, PR 130, 1605 (1963)
C. M. Herdman, P. N. Roy, R. G. Melko, and A.D., PRB B 94, 064524 (2016)

Scaling of Spatial Entanglement

Expected scaling result for 1d critical systems:

$$S_\alpha(\ell, L) = \frac{c}{6}\left(1 + \frac{1}{\alpha}\right)\log\left[\underbrace{\frac{L}{\pi}\sin\left(\frac{\pi\ell}{L}\right)}_{D(N,\ell)}\right] + c_\alpha + O\left(\frac{1}{\ell^{p_\alpha}}\right)$$





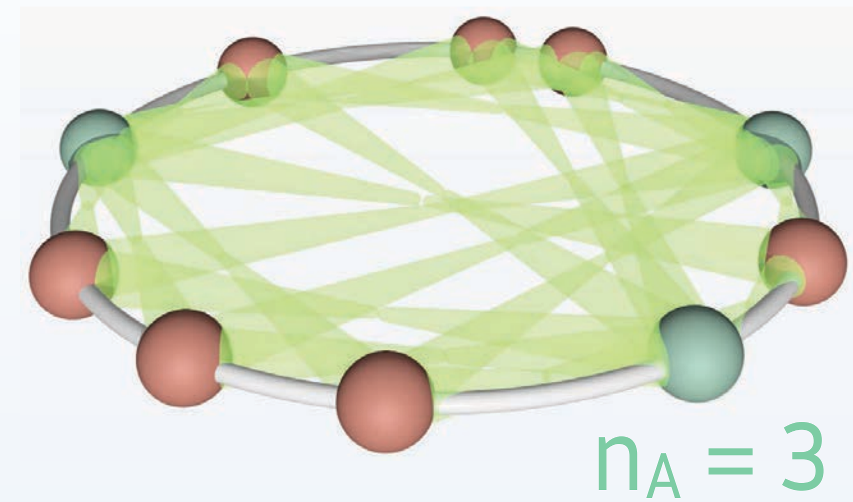
$c \simeq 1$

P. Calabrese and J. Cardy, J. Stat. Mech. P06002 (2004)
J. Cardy and P. Calabrese, J. Stat. Mech. P04023. (2010)
C. M. Herdman, P. N. Roy, R. G. Melko, and A.D., PRB B 94, 064524 (2016)

Particle Entanglement

Predicted general scaling form by Haque & Schoutens:

$$S(n, N) = a \ln \binom{N}{n} + b$$



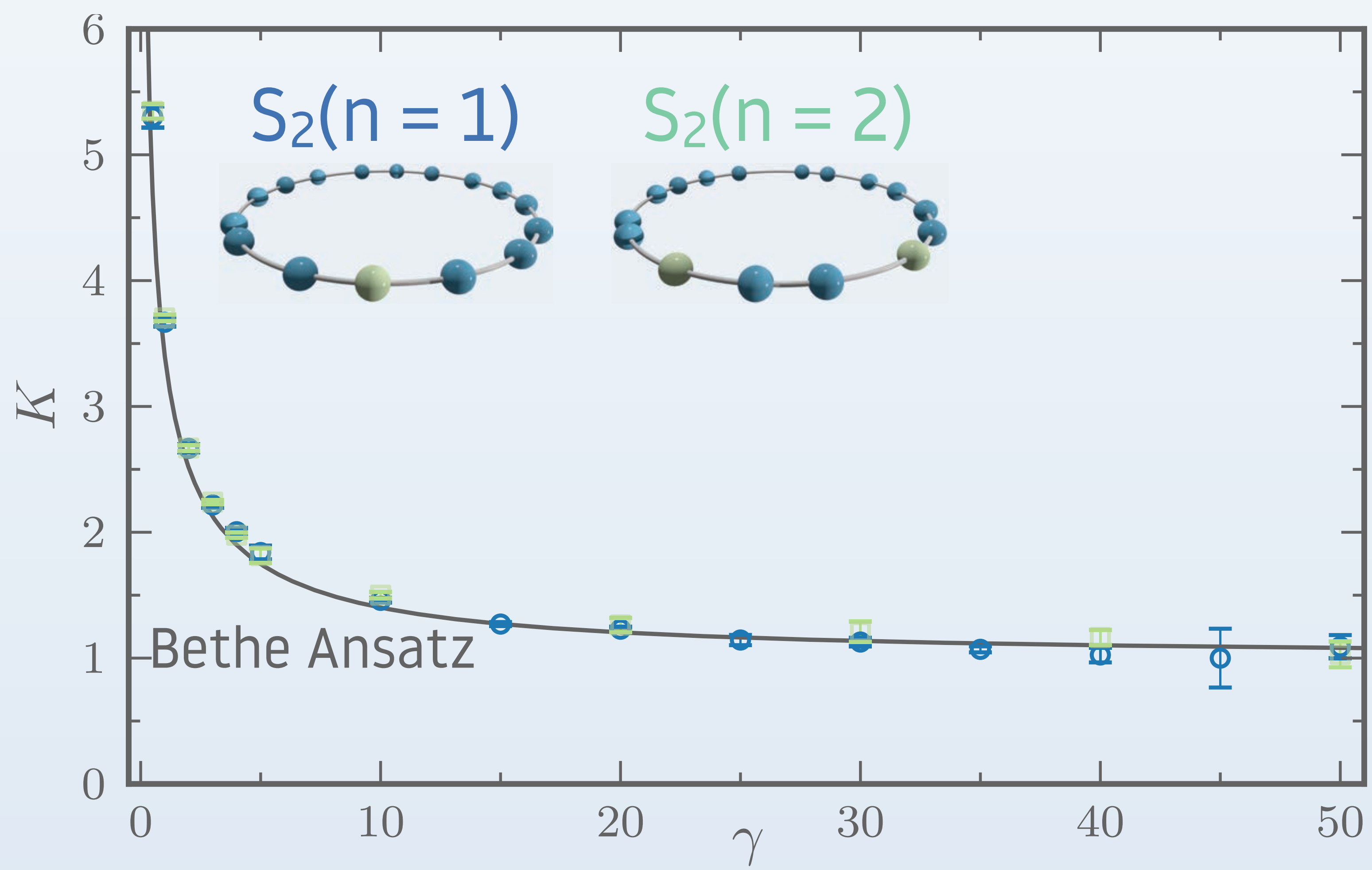
O. Zozulya, M. Haque, and K. Schoutens, PRA 78, 042326 (2008)
M. Haque, O. S. Zozulya, and K. Schoutens, J. Phys. A 42, 504012 (2009)

For a bosonic Luttinger Liquid: $a = n/K$

C. Herdman and A.D., Phys. Rev. B, 91, 184507 (2015)

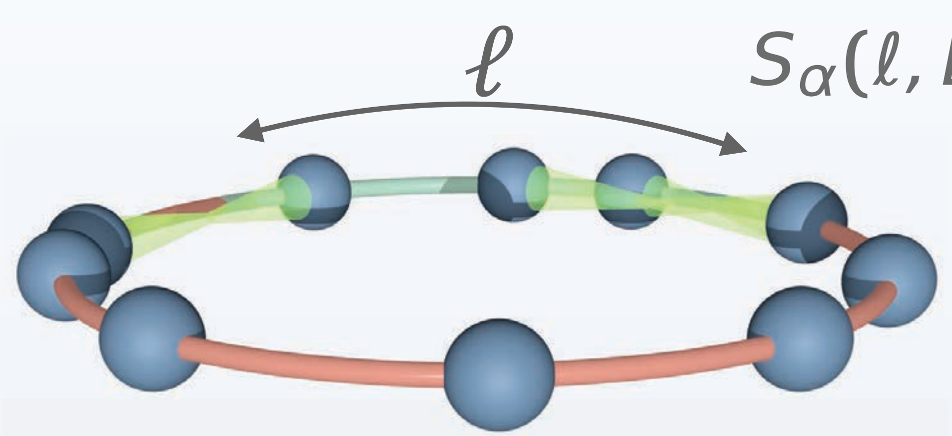
confirmation with QMC

Luttinger parameter



Sensitivity of Entanglement to Statistics

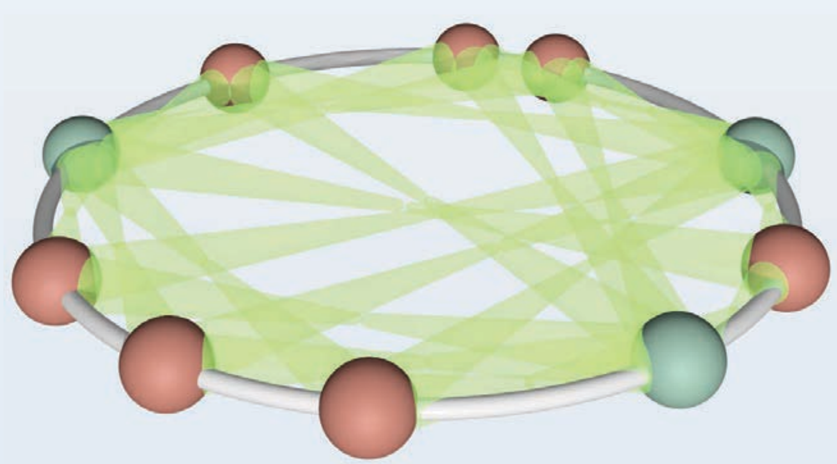
spatial bipartition: 1d critical systems



$$S_\alpha(\ell, L) = \frac{c}{6}\left(1 + \frac{1}{\alpha}\right)\log\left[\frac{L}{\pi}\sin\left(\frac{\pi\ell}{L}\right)\right] + c_\alpha + O\left(\frac{1}{\ell^{p_\alpha}}\right)$$

P. Calabrese and J. Cardy, J. Stat. Mech. P06002 (2004)
J. Cardy and P. Calabrese, J. Stat. Mech. P04023. (2010)

particle bipartition:



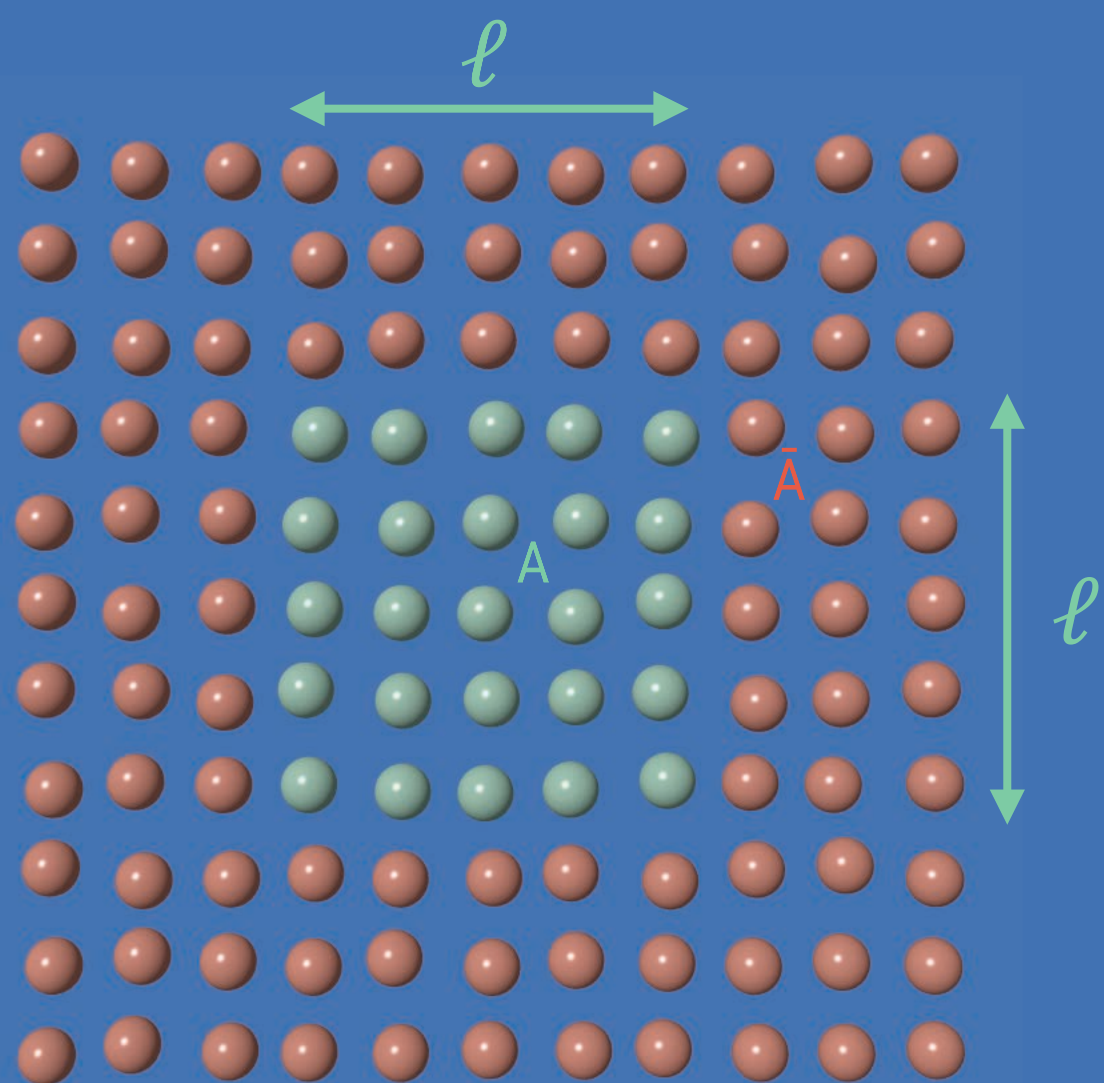
$$S_\alpha(n, N) = a_\alpha(n)\ln\binom{N}{n} + b_\alpha(n) + \mathcal{O}\left(\frac{1}{N^{\gamma_\alpha(n)}}\right)$$

a = 1 for fermions
= n/K for 1d bosons

C. Herdman and A.D., Phys. Rev. B, 91, 184507 (2015)

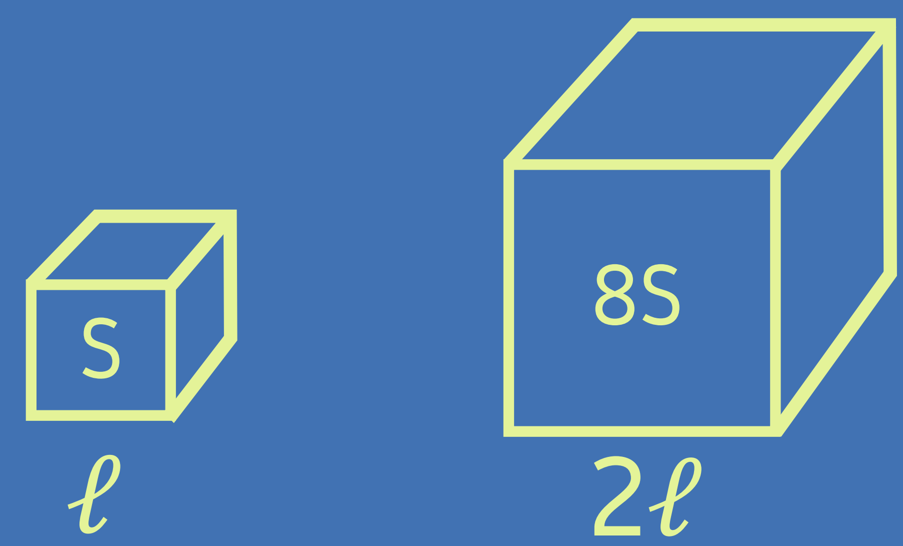
H. Barghathi, E. Casiano-Diaz, and AD, JSTAT. 2017, 083108 (2017)
M. Haque, O. S. Zozulya, and K. Schoutens, J. Phys. A 42, 504012 (2009)

How does entanglement scale with the size of the subregion in d>1?



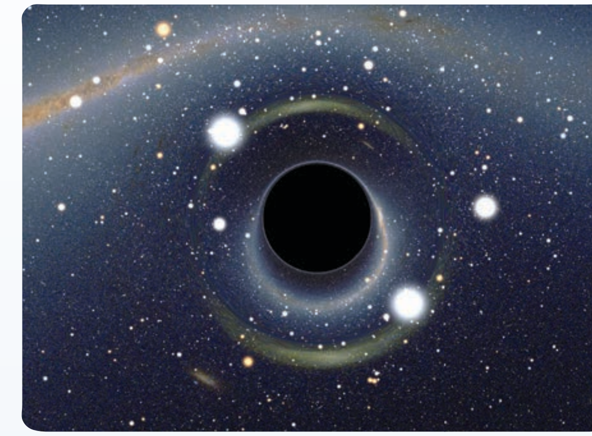
$$S(\ell) \sim \ell^{\lambda}$$

thermodynamic entropy is extensive ⇒ $\lambda = d$



entanglement area law?

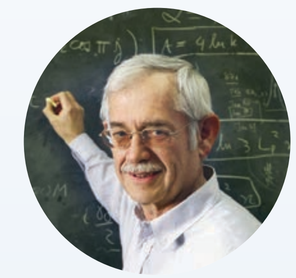
Black Hole Entropy Area Law



Black hole thermodynamics:

- Quantum black holes emit thermal radiation
- Area Law: entropy of a black hole is proportional to surface area, not volume!

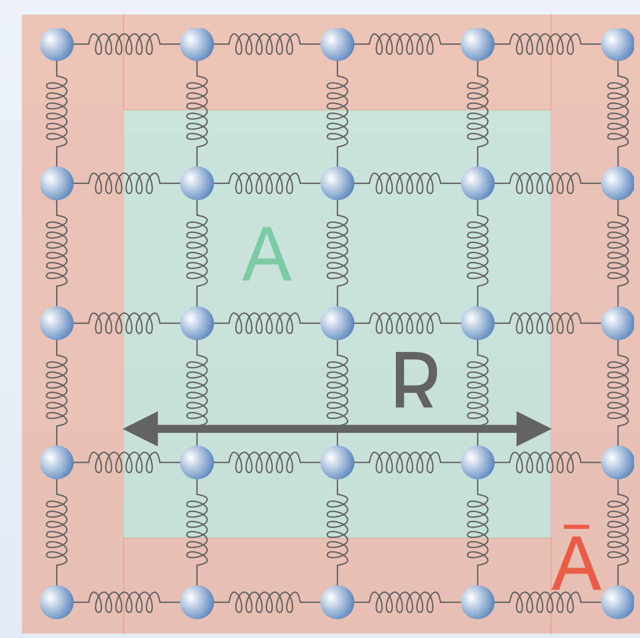
$$S_{BH} \propto \text{area}$$



J.D. Bekenstein, *PRD* 7, 2333 (1973)
S.W. Hawking, *Nature* 248, 30 (1974)

Is this due to entanglement?

- Toy model: coupled harmonic oscillators
- Area Law: number of springs connecting A with Ā scales with boundary size



M. Srednicki Phys. Rev. Lett. **71**, 666 (1993)

General "Derivation" of the Area Law

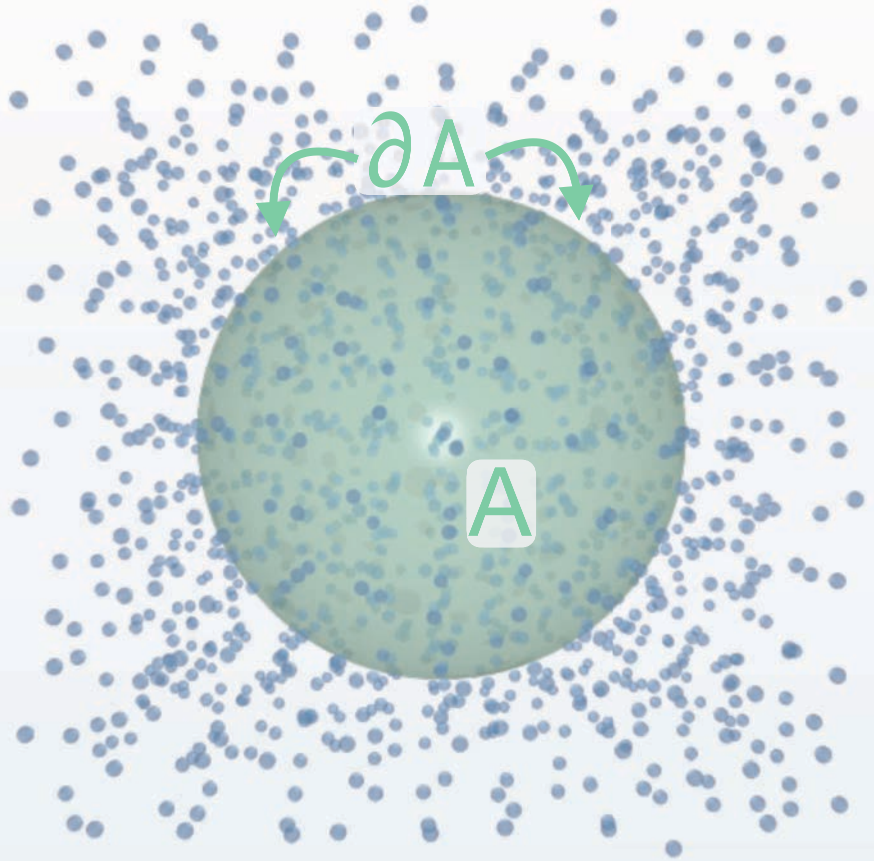
Based on 2 physical principles:

1. S(A) arises from correlations local to the entangling surface

at scale r: $$S(A,r) = \int_{\partial A} \frac{ds}{r^{d-1}} g(\partial A, r)$$

(d-1) surface element

local quantity depending on curvature of ∂A



2. All length scales contribute: microscopic to macroscopic

$$S(A) = \int_{r_0}^{R} d(\log r) S(A,r)$$

spherical ∂A $\Rightarrow g = c_0 + c_1\left(\frac{r}{R}\right)^2 + c_2\left(\frac{r}{R}\right)^4$

no odd powers

$$= \int_{r_0}^{R} \frac{dr}{r} \int_{\partial A} \frac{ds}{r^2}\left[c_0 + c_1\left(\frac{r}{R}\right)^2 + \cdots\right]$$

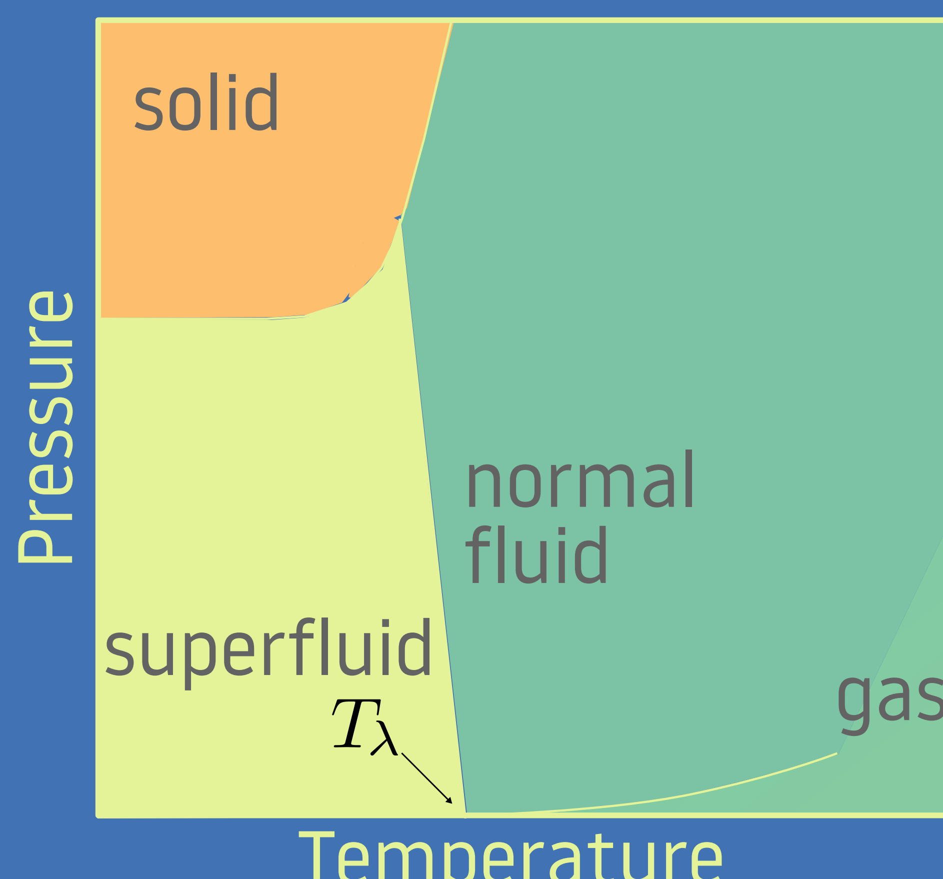
$$= \frac{c_0}{2}\left(\frac{R}{r_0}\right)^2 + c_1 \log\frac{R}{r_0} + const. + O\left(\frac{1}{R^2}\right)$$

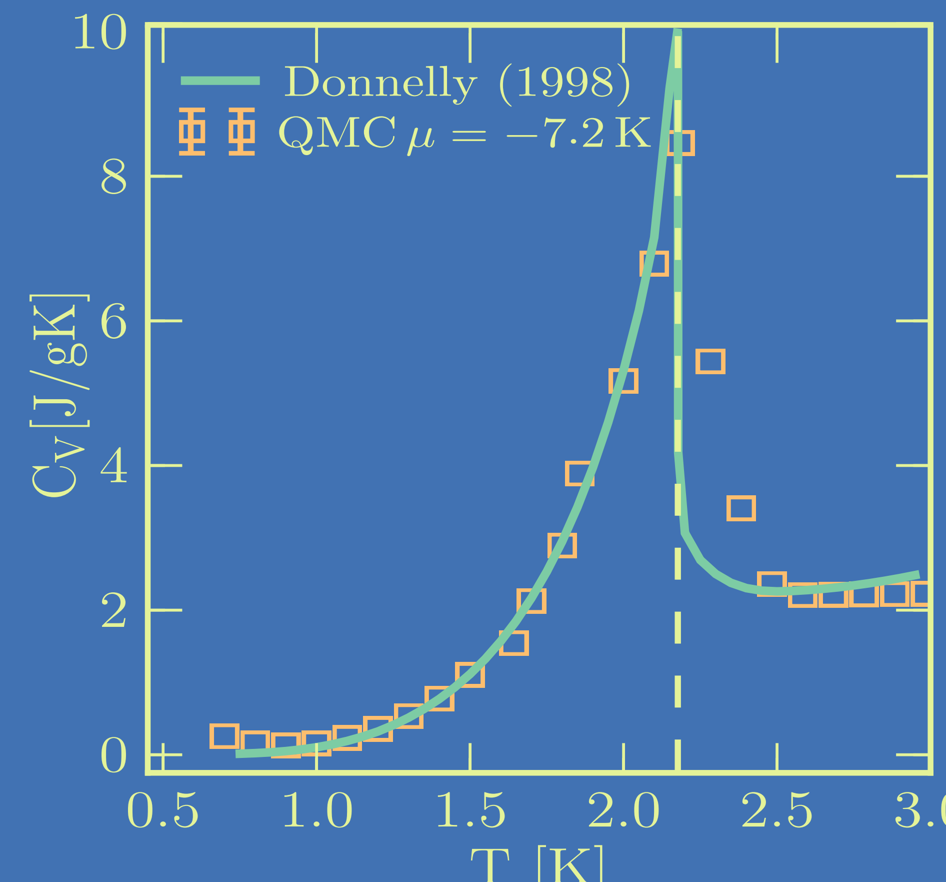
area law for a sphere!

M.B. Hastings, *J. Stat. Mech.*, P08024 (2007)
S.N. Solodukhin, Phys. Lett. B, 693, 605 (2010)
B. Swingle, arXiv:1010.4038
H. Liu and M. Mezai, JHEP 1304, 162 (2013)
L. Hayward Sierens, Ph.D. Thesis, (2017)

What about a real quantum phase of matter?

helium-4

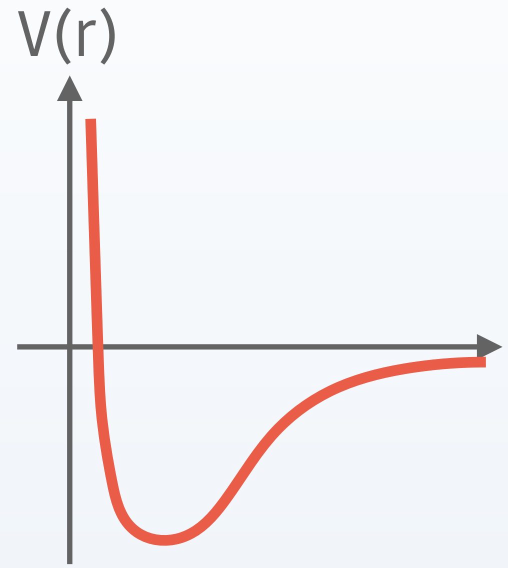


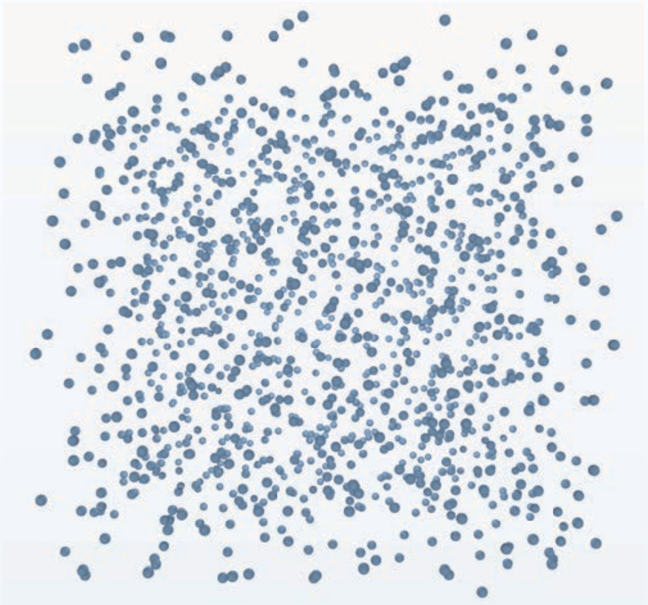


Entanglement in Superfluid ^4He

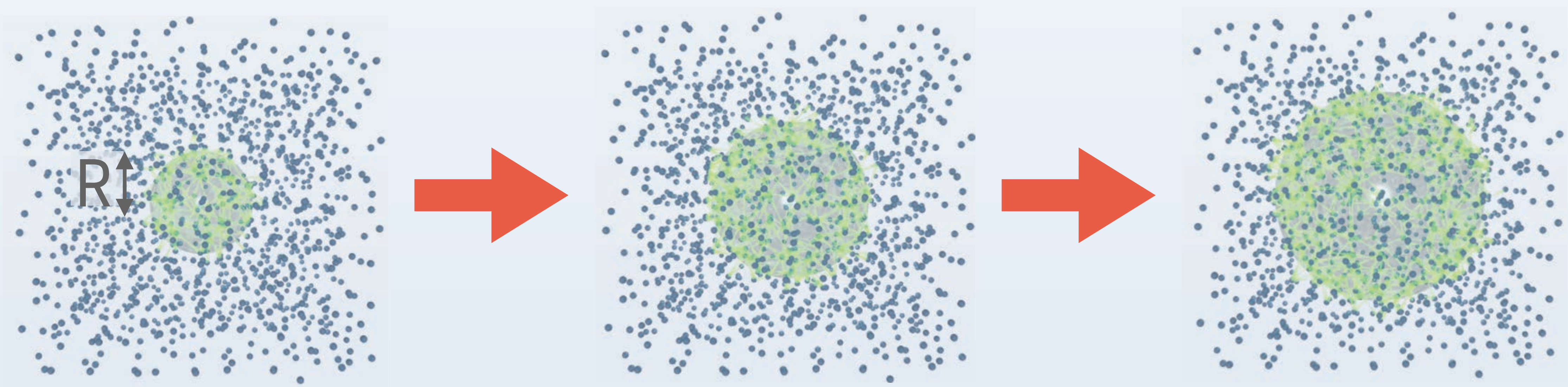
3d box at T = 0 with periodic boundary conditions at SVP

$$\hat{H} = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \hat{\nabla}_i^2 + \sum_{i=1}^{N} \hat{\mathcal{V}}_i + \sum_{i<j} \hat{\mathcal{U}}_{ij}$$





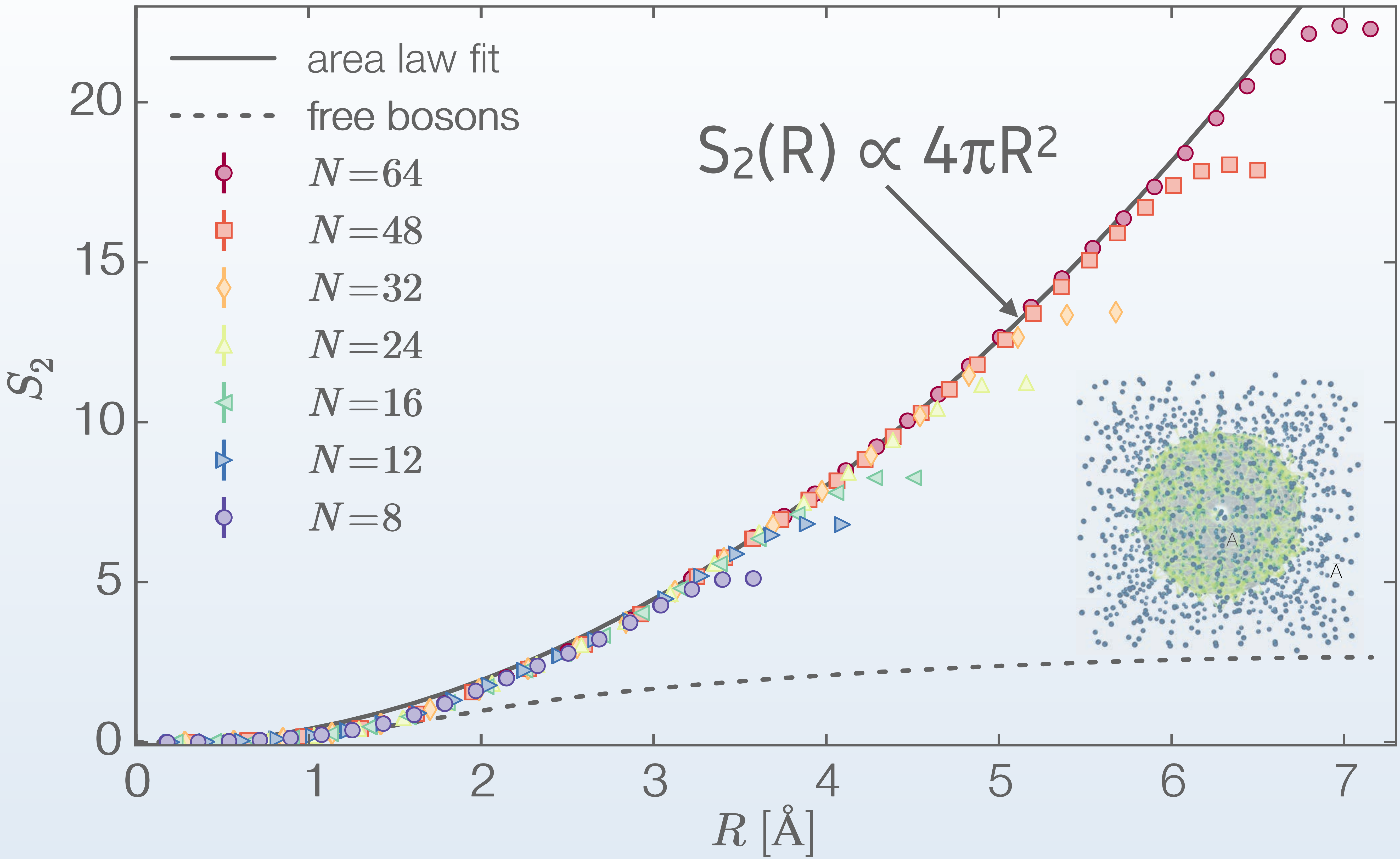
Measure entanglement $S_2(R)$ between spherical region of radius R and the rest of the box



Investigate scaling by changing the radius of the sphere

C. M. Herdman, P.-N. Roy, R. G. Melko & A.D. Nature Phys 13, 556 (2017)

Scaling of the Entanglement



C. M. Herdman, P.-N. Roy, R. G. Melko & A.D. Nature Phys 13, 556 (2017)

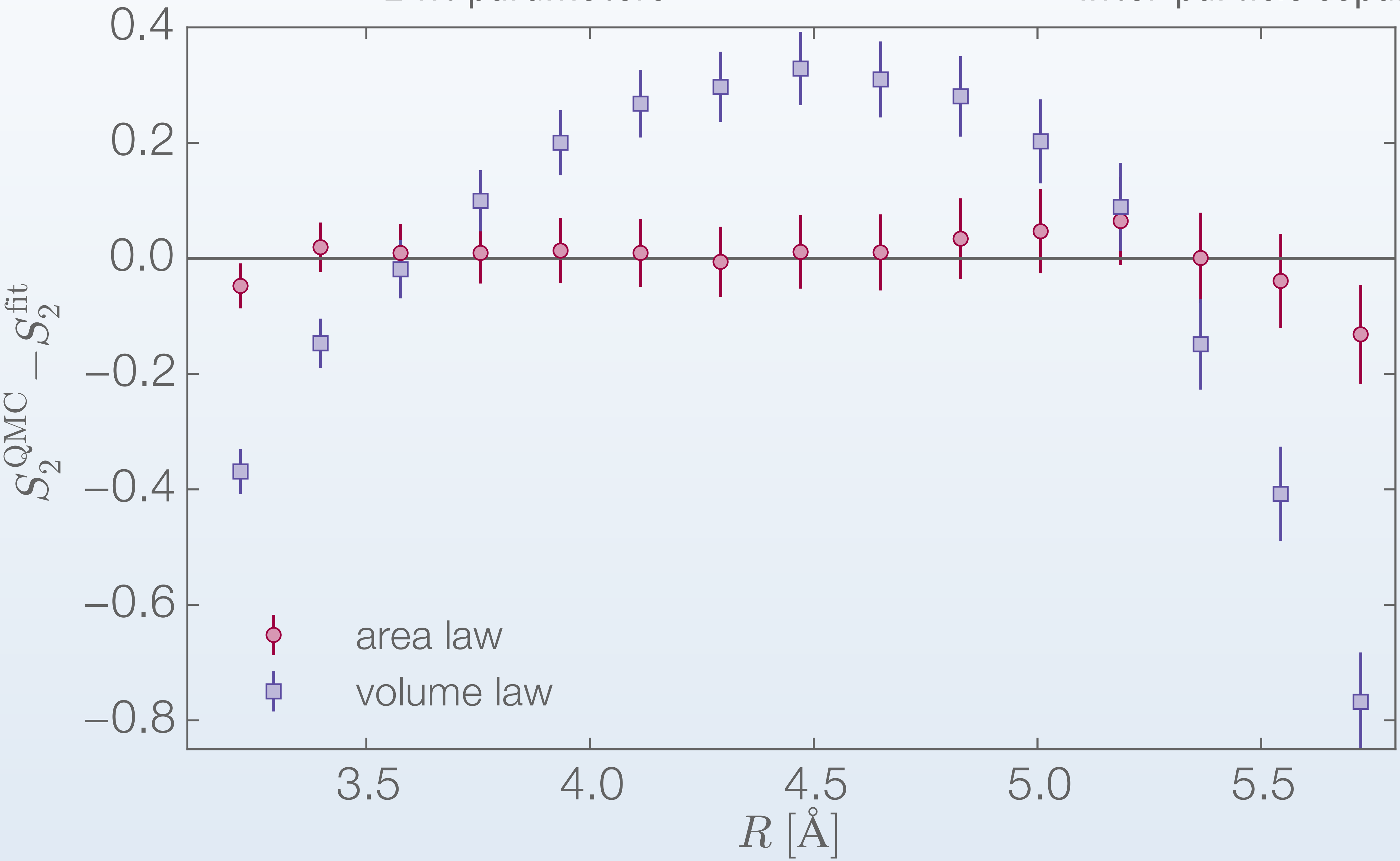
Area Law vs. Volume Law

Area: $S_2^{fit} = 4\pi a \left(\frac{R}{r_0}\right)^2 + c$

Volume: $S_2^{fit} = \frac{4\pi}{3} a \left(\frac{R}{r_0}\right)^3 + c$

2 fit parameters

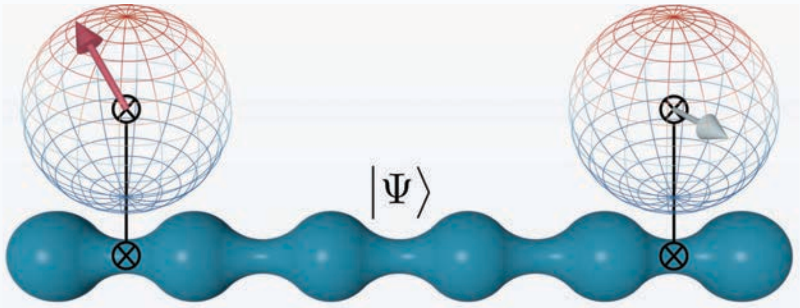
inter-particle separation



C. M. Herdman, P.-N. Roy, R. G. Melko & A.D. Nature Phys 13, 556 (2017)

Entanglement in quantum liquids can be useful

Physical constraints determine the entanglement that can be transferred to a register for quantum information processing



Discovery of an area law in a real quantum liquid

Quantum entanglement scales with the surface area and not volume in superfluid ^{4}He

$$S_2 \propto R^2$$

Analogous to Bekenstein-Hawking black hole entropy

Partners in Research & Computing

XSEDE

Extreme Science and Engineering
Discovery Environment

VACC

VERMONT ADVANCED COMPUTING CORE

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